Two planes on merging routes are:
--same distance from the intersection
--traveling at different speeds.

A separation distance is required at the intersection.

SMART SKIES

Airspace Systems—Predicting Air Traffic Conflicts

Teacher Guide

Curriculum Supplement 5

Overview of Curriculum Supplement 5

You may choose to spread the experiment and calculation activities over two or three class periods, allowing time for setting up the experiment, conducting the experiment, doing the calculations, and discussing the outcomes.

This is the fifth in a series of Airspace Systems Curriculum Supplements that address distance-rate-and time problems. Each Curriculum Supplement consists of an experiment, worksheets to support the experiment, worksheets for paper-and-pencil calculations, a student analysis of the airspace scenario, and optional pre- and post-assessment instruments.

In this Curriculum Supplement, the controller must merge two flows of air traffic into one stream. The planes are each the same distance from the intersection and each is traveling at a different constant (fixed) speed. For safety reasons, when the first plane reaches the intersection, the planes must be separated by a distance greater than or equal to a given standard separation distance. If their separation is less than this standard, a separation violation will occur.

Of the eight *Airspace Systems* Curriculum Supplements, this scenario is two steps removed from the simplest case (Curriculum Supplement 1) in which the speeds, as well as the distances, are the same. As in Curriculum Supplements 3 and 4, there is a separation requirement. This is the first Curriculum Supplement in which the planes travel at *different* speeds.

The current scenario builds upon the simpler case set forth in Curriculum Supplement 1 in which the planes are also the same distance from the intersection but are traveling at the same constant speed. In that case, the planes arrived at the intersection at the same time. However, Curriculum Supplement 1 did not require a separation standard.

There are basically two ways to avoid a predicted separation violation: change the speed or change the route of one of the aircraft. The previous Supplement (Curriculum Supplement 4) used an alternate route to try to resolve the separation violation. The current Supplement (Curriculum Supplement 5) changes the speed of one of the aircraft.

Airspace Scenario

Students will determine if two airplanes traveling on different merging routes will obey the separation standard at the intersection of their flight routes.

The airplanes are each **the same distance from the point of intersection**.
The airplanes are traveling at **different constant (fixed) speeds**. There is also a **separation requirement**. That is, there is a requirement for a minimum distance between the planes when the first plane arrives at the point where the two routes meet.

Objectives

Students will determine the following:

> If two planes are traveling at different speeds on two different routes and the planes are each the same distance from the point where the two routes come together, the planes will arrive at the intersection at different times.
>
> The separation distance is proportional to the difference in speeds. If the difference in speeds were twice as great, then the separation distance at the intersection would also be twice as great.

Introducing Your Students to the Airspace Scenario

If you have not already done so, you may want to show the "Gate to Gate" CD-ROM to introduce your students to the air traffic control system. (For more detail, see the Smart Skies Airspace Systems Introduction for Teachers.)

To help your students understand the problem, you can ask them to consider this related problem that is set in a more familiar context:

Two students, Ana and Alex, plan to meet at the movies. Ana lives 20 blocks from the theater. Alex also lives 20 blocks from the theater. Ana and Alex will each leave their homes at the same time and walk at different constant (fixed) speeds. Alex walks more slowly than Ana.

You can ask your students these questions:

Who will arrive at the theater first, Ana or Alex? Why?

Activity 5.0 --

Problem Statement

In a real-world scenario, one plane's speed might be 400 nautical miles per hour and the other plane's speed might be 320 nautical miles

Problem Statement
Worksheet 5.0 describes and illustrates the airplane scenario. The speed of one airplane is 1/2 foot/second. The speed of the other airplane is 1/3 foot/second. Both airplanes are 20 feet from the point of intersection. The separation standard is 5 feet along the flight path. In particular, at the point of intersection, the planes must be at least 5 feet apart.

per hour. Each plane might be 40 nautical miles from the point of intersection.
The separation standard might be 5 nautical miles.

An international nautical mile is 1,852 meters.

A nautical mile per hour is called a "knot".

As a problem extension, you may want to ask your students to solve the problem using real-world data.

Student Handout:
Worksheet 5.0

Note: These speeds and distances were chosen to reflect the classroom experiment that the students will conduct and are not related to real-world parameters.

Five questions are posed:

Q1: How many seconds will it take Flight WAL27 to travel 20 feet to the point where the routes come together?

Q2: How many seconds will it take Flight NAL63 to travel 20 feet to the point where the routes come together?

Q3: Will the planes meet at the point where the routes come together?

Q4: If not, how far apart will the planes be when the first plane reaches the point where the routes come together?

Q5: Does this distance meet the separation requirement?

Since the planes are traveling at different constant (fixed) speeds and each must travel the same distance to the point of intersection, students may be able to answer Question 3 correctly without answering Questions 1 and 2. That is, they may realize that the planes will not meet.

To answer Questions 4 and 5, students must calculate or graph to determine the number of seconds for the faster plane to reach the intersection and the number of feet between the two planes at that moment.

Materials
Worksheet 5.0: Problem Statement

Activity 5.1 -- Pretest

Estimated time:
15 - 30 minutes

The pretest is ***optional.***

Instead of distributing the pretest, you may want to use the questions to guide a classroom discussion.

Pretest—Make a Prediction
The pretest steps the student through a careful reading of the airplane problem statement. The student is then asked to predict the outcome of the given airplane scenario.

The pretest may be assigned as either an individual or a small-group activity.

If your students have completed other Airspace Systems Curriculum Supplements, you may want to direct them to use a particular calculation method or methods to answer the pretest

Student Handout:
Worksheet 5.1A
Worksheet 5.1B

questions. In that case, Worksheet 5.1B contains blank vertical line plots as well as grids that students can use as they do their calculations.

Materials
Worksheet 5.1A: Pretest—Make A Prediction
Worksheet 5.1B: Lines and Grids

Activity 5.2 -- Experimentation

Estimated time:
Setup—30 minutes
Experiment—30 minutes

*For a step-by-step student orientation to the Experiment, see Curriculum Supplement 0, the introduction to **Airspace Systems**.*

Student Handouts:
Worksheet 5.2A
Worksheet 5.2B
Worksheet 5.2C

You may want to give students an overview of the experiment including an explanation of what they will do in each activity.

You may want to ask your students to compare the experiment distances and speeds with the real-world speeds given in the sidenote for Activity 5.0.

You may want to ask your students to estimate the route layout before they measure.

Classroom Experiment
In this small-group activity, students mark off the jet routes on the classroom floor or on an outdoor area. Students assume the roles of pilots, air traffic controllers, and NASA scientists. The pilots step down the jet routes at a prescribed pace. The NASA scientists track and record the pilots' times and the pilots' distances from the intersection of the routes. The air traffic controllers set the pace and measure the separation distance when the first plane arrives at the intersection.

Materials
Activity 5.2A: Set Up the Experiment
--sidewalk chalk or masking tape
--measuring tape or ruler
--marking pens (optional)

Activity 5.2B: Conduct the Experiment
--1 stopwatch or 1 watch with a sweep second hand or 1 digital watch that indicates seconds
--pencils and Data Sheets (Worksheet 5.2C)
--signs identifying pilots, controllers, and NASA scientists
Note: the signs are available on the Smart Skies website.
--clipboard (optional)

Student Handouts:
--Worksheet 5.2A: Set Up the Experiment
--Worksheet 5.2B: Conduct the Experiment
--Worksheet 5.2C: Data Sheet

Worksheet 5.2A, Set Up the Experiment

If there is not enough room to set up two 20-foot routes at right angles to one another, another angle may be used. As an alternative, the routes may be set up parallel to each other.

(Caution: parallel routes may confuse students who have not had much experience with the experiment. They may not make the connection between the parallel routes and the given merging routes.) In any case, allow enough distance between the routes so that the two pilots are not distracted by one another.

Students who have little experience in measurement may benefit from first practicing skip counting (by 6 and by 4) to prepare them to measure 6-inch lengths and 4-inch lengths.

You may want to set up one pair of jet routes as a model that your students can copy.

After a group of students has completed its jet route set-up, you may find it helpful to have them compare their work with another student set-up.

Worksheet 5.2B, Conduct the Experiment

Assign students to positions on 6-8 person teams as follows:

--Lead Air Traffic Controller (1 student)
--Secondary Air Traffic Controller (1 student)
--Pilots (2 students)
--NASA Scientists, 1 or 2 for each plane (2 – 4 students)

It may be difficult for some student pilots to take 6-inch or 4-inch steps by placing one foot in front of the other. Instead, advise the pilots to place one foot on either side of the jet route and align their toes at each mark. It may be helpful for students to practice.

After the jet routes are set up, have one group of students demonstrate the experiment while the rest of the class observes. Discuss and address any issues that may arise.

Perform the activity at least three times. Compare the results of each trial. Discuss the validity of the results.

Extensions:

1. Repeat the activity using different students as the Air Traffic Controllers, Pilots, and NASA Scientists.

2. Repeat the activity using jet routes longer than 20 feet. Increase the 5-foot separation standard using the same percent increase used for the jet route lengths. Increase the plane speeds and the step sizes to 1 foot/second and 2/3 foot/second, respectively.

3. Have students draw a scale model of the experiment using real-world data. (See the sidenote for Activity 5.0).

Activity 5.3 -- Calculations

Calculate the Time for Each Plane to Reach the Intersection
This activity presents six different methods students can use to determine the number of seconds for the faster plane to arrive at

Estimated time:
15 - 30 minutes per worksheet

the intersection point and the distance between the two planes at that moment.

Each worksheet may be assigned as either an individual or a small-group activity.

You can choose to assign one, some, or all of the worksheets. If students have completed an earlier Curriculum Supplement, you may decide to focus on only one worksheet.

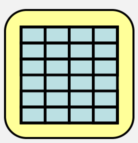
You may want to assign some worksheets before and some worksheets after the experiment.

The calculation methods range in order of difficulty as follows:

- Counting (completing a table)
- Drawing blocks to make a bar graph
- Plotting points on two vertical lines
- Plotting points on a Cartesian coordinate system
- Deriving and using the distance-rate-time formula
- Graphing two linear equations

Worksheet 5.3A, Count Feet and Seconds

Student Handout:
Worksheet 5.3A

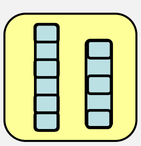


Students use patterns and skip-counting to complete a table and solve the problem. At the end of this activity, students may realize it is faster to multiply than to add to obtain the answer.

Prerequisite skills: count by 2s, count by 3s.

Worksheet 5.3B, Draw Blocks

Student Handout:
Worksheet 5.3B



Students draw blocks, each representing the distance each plane travels in 10 seconds. The students "stack" their blocks along two vertical number lines (one line for each plane) that represent the jet routes.

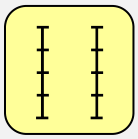
Notice that the vertical lines are numbered from 20 at the bottom to 0 at the top. Students begin to stack the blocks at the starting point of each plane, 20 feet away from the intersection. The intersection of the routes is represented with 0 at the top of each number line.

To help students make the connection between the "inverted Y" jet routes and the vertical scales, students are first asked to plot a point on the original jet route diagram and then stack the corresponding block along the vertical scale.

Prerequisite skills: read and build a bar graph with a vertical scale marked in 1-foot units; count by 10s.

Worksheet 5.3C, Plot Points on Two Vertical Lines

Student Handout:
Worksheet 5.3C



This graph is similar to the way families record and compare the height of their children at the same ages. They mark off each child's birthday height (distance from the floor) on a doorway and then record their age (time since birth) beside the height mark.

The students plot their points along two vertical number lines (one line for each plane) that represent the jet routes.

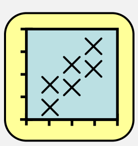
Notice that the vertical lines are numbered from 20 at the bottom to 0 at the top. The bottom of each number line corresponds to the starting point of each plane, 20 feet away from the intersection. The intersection of the routes is represented with 0 at the top of each number line.

To help students make the connection between the "inverted Y" jet routes and the vertical scales, students are first asked to plot a point on the original jet route diagram and then plot the corresponding point on the vertical scale.

Prerequisite skills: plot a point on a (vertical) number line.

Worksheet 5.3D, Plot Points on a Cartesian Coordinate System

Student Handout:
Worksheet 5.3D



Notice that the vertical axis is numbered from 0 at the top to **negative** 20 at the bottom. The numbers along the vertical axis represent the distance (with a negative sign attached) from the point where the two routes meet. Negative numbers are used because the points lie below the horizontal axis (the horizontal line at 0 feet).

Prerequisite skills: plot a point on a Cartesian coordinate system (the xy-plane)

Extension (optional):
For each plane, connect the points with a straight line. Find the equation of each line.

Worksheet 5.3E, Derive the Distance-Rate-Time Formula

Student Handout:
Worksheet 5.3E

d = r • t

Students use patterns to derive the distance-rate-time formula in the form d = rt.

Prerequisite skills:
Use patterns to make a generalization.

Worksheet 5.3F, Use the Distance-Rate-Time Formula

Student Handout:
Worksheet 5.3F

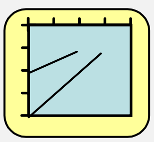
t = d / r

Students apply the distance-rate-time formula in the form t = d/r.

Prerequisite skills:
Substitute numbers into a formula.

Worksheet 5.3G, Graph Two Linear Equations

Student Handout:
Worksheet 5.3G



Notice that the points are plotted in the fourth quadrant. So the given portion of the y- axis is numbered from 0 at the top to **negative** 20 at the bottom. The numbers along the y-axis represent the distance (with a negative sign attached) from the point where the two routes meet. Negative numbers are used because the points lie below the x-axis.

Prerequisite skills:
Graph a linear equation by making a table of ordered pairs.
Find the slope of a line given the equation of the line and the graph of the line.

Caution: Students may confuse the path of a plane with the graph of the plane's distance from the intersection as a function of time. In particular, the intersection of the graphs at t=0 indicates that each plane is the same distance, 20 feet, from the intersection of the routes.

Extension (optional):
You may want to ask your students to find the intercepts of each line and interpret those intercepts in the context of the airspace problem.

For each plane, the y-intercept represents the plane's initial distance (with negative sign attached) from the intersection point.

For each plane, the x-intercept represents the number of seconds for the plane to reach the intersection point.

The horizontal distance between the x-intercepts represents the number of seconds between the arrival of the two planes at the intersection.

Materials

Worksheet 5.3A: Calculate the time—count feet & seconds
Worksheet 5.3B: Calculate the time—draw blocks
Worksheet 5.3C: Calculate the time—plot on two vertical scales
Worksheet 5.3D: Calculate the time—plot points on a Cartesian coordinate system
Worksheet 5.3E: Derive the Distance-rate-time formula
Worksheet 5.3F: Use the Distance-rate-time formula
Worksheet 5.3G: Graph Two Linear Equations

Activity 5.4 --

Analysis

Estimated time:
45 minutes

Student Handout:
Worksheet 5.4

Compare the Experimental Results with the Predicted Results
Students compare the outcome of the experiment with their pretest predictions.

This activity may be assigned as either an individual or a small-group activity.

If you assigned some calculation worksheets (Activity 5.3) prior to the experiment, students can compare their calculations with the experimental results.

You may want to assign some Activity 5.3 calculation worksheets after the experiment to give students another basis for comparison.

As part of the Analysis, you may also want to ask your students to create a similar problem in a different setting. They have already considered a problem in which two students walk from their respective homes to a movie theater. (See the Airspace Scenario section of this document.)

Now, you might suggest they consider two cars traveling in parallel lanes on the same road, with the two lanes merging into one lane. Each car is traveling at a different constant (fixed) speed. The cars are each the same distance from the merge. Students should realize that the cars will arrive at the merge at different times.

Note: To be consistent with the airspace scenarios, it is important that for each problem created by you or your students, you choose a fixed (constant) speed for each vehicle or person. (For example, a rocket launch scenario would *not* be appropriate because a launched rocket typically accelerates and therefore its speed is not constant.)

Materials

Worksheet 5.4: After the Experiment

Activity 5.5 --

Posttest

Estimated time:
15 - 30 minutes

*The posttest is **optional.***

Student Handouts:
Worksheet 5.5
Worksheet 5.1B

Curriculum Supplement Posttest

This activity may be assigned as either an individual or a small-group activity.

You can direct your students to use a particular calculation method or methods to answer the posttest questions. Worksheet 5.1B (used for the Pretest) contains blank vertical line plots and grids that students can use as they do their calculations.

Materials
Worksheet 5.5: Posttest
Worksheet 5.1B: Lines and Grids

Name

Problem Statement

In the picture below, two airplanes are flying on different routes.

The World Airlines plane has flight number **WAL**27.

The speed of Flight WAL27 is 1/2 foot/second (0.15 meters/second).

The National Airlines plane has flight number **NAL**63.

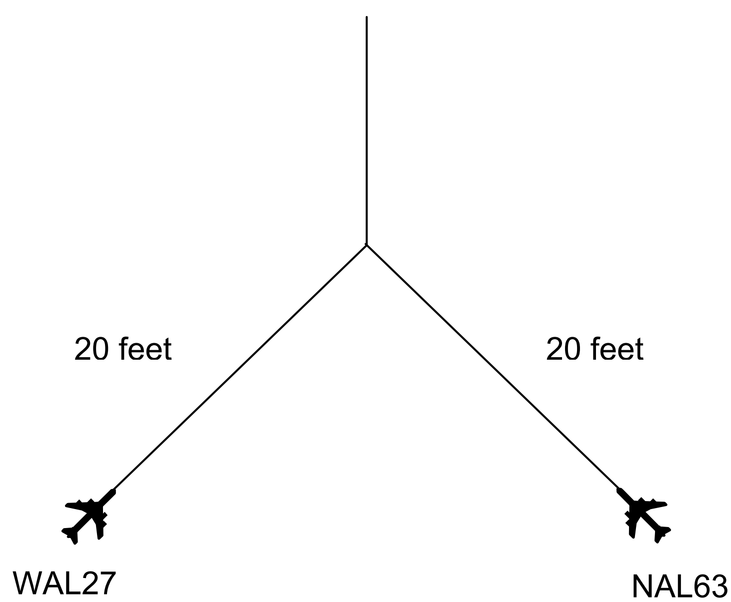
The speed of Flight NAL63 is 1/3 foot/second (0.10 meters/second).

Flight WAL27 is 20 feet (6.1 meters) away from the point where the two routes come together.

Flight NAL63 is 20 feet (6.1 meters) away from the point where the two routes come together.

To keep the pilots and passengers safe, the planes must be at least 5 feet (1.5 meters) apart when the first plane reaches the point where the routes come together. If the distance between the planes is less than 5 feet, then it may not be safe.

This 5-foot distance is called a *separation standard*.



____
Name

Question 1: How many seconds will it take Flight WAL27 to travel 20 feet to the point where the two routes come together?

Question 2: How many seconds will it take Flight NAL63 to travel 20 feet to the point where the two routes come together?

Question 3: Will the planes meet at the point where the two routes come together?

Question 4: If not, how many feet apart will the planes be when the first plane reaches the point where the routes come together?

Question 5: In Question 4, does the distance meet the 5-foot separation standard?

____
Name

Pretest—Make a Prediction

In the picture on the next page, two airplanes are flying on different routes.

1. Draw a circle around the point where the routes come together.

The World Airlines plane has flight number WAL27.
The speed of Flight WAL27 is 1/2 foot/second.

2. Write the speed of Flight WAL27 next to its picture.

3. How far does Flight WAL27 travel in one second? ________________

The National Airlines plane has flight number NAL63.
The speed of Flight NAL63 is 1/3 foot/second.

4. Write the speed of Flight NAL63 next to its picture.

5. How far does Flight NAL63 travel in one second? ________________

6. Do you think that the two planes will meet at the point where the two routes come together? ________________

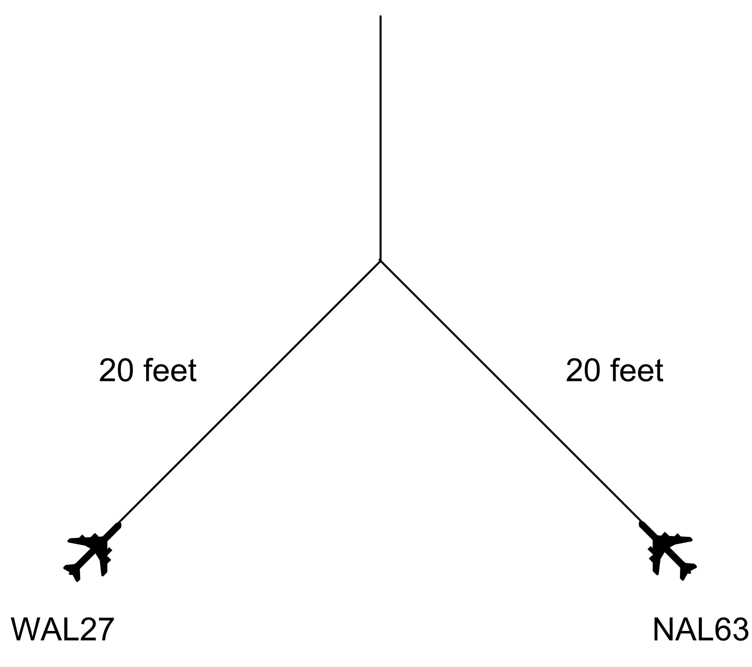
   Why or why not? ____

   ____

____
Name

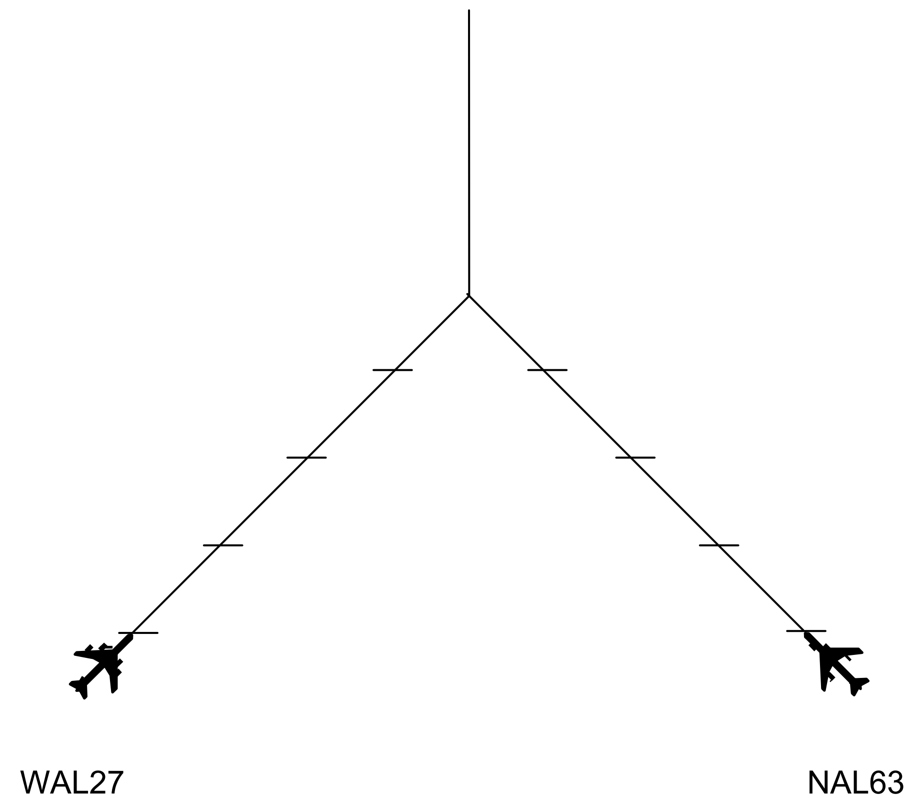
7. If not, how many feet apart do you think the planes will be when the first plane reaches the point where the routes come together? ________________

8. Does your answer to Question 7 meet the 5-foot separation standard? That is, do you think the planes will be at least 5 feet apart when the first plane reaches the point where the routes come together? ________________



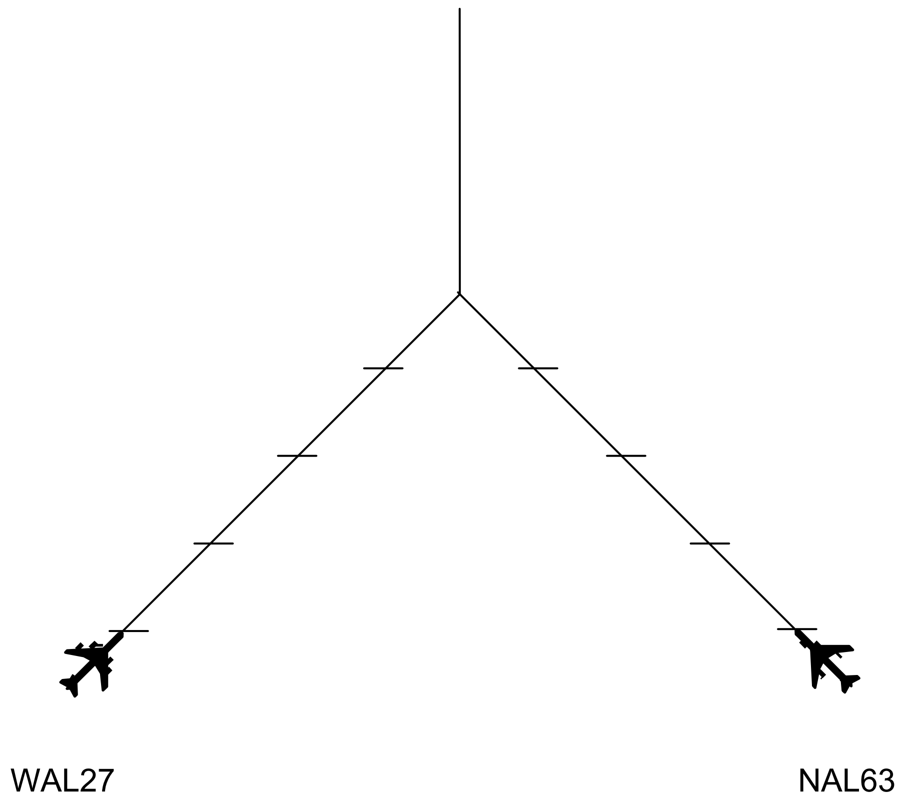
Name

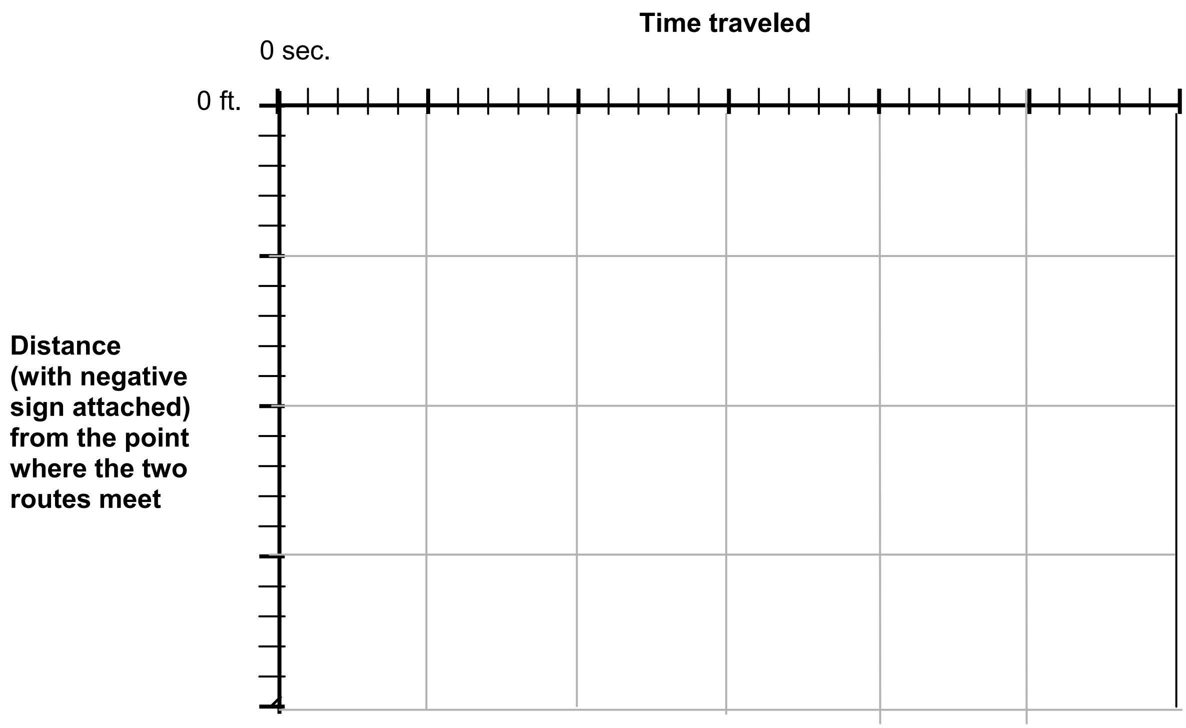
Lines and Grids



Name

Lines and Grids





____
Name

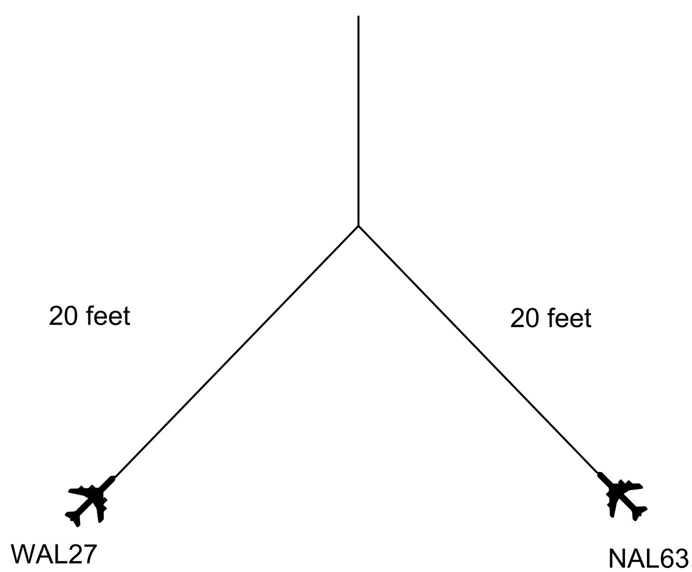
Set Up the Experiment

1. Use sidewalk chalk (or masking tape) to lay out 2 jet routes, one for each airplane as shown below.
   Each route should be 20 feet long.

2. The speed of Flight WAL27 is 1/2 foot/second. Stand at the beginning of the jet route for Flight WAL27. Place a mark (or a piece of masking tape) every 1/2 foot (every 6 inches) along the jet route. This will guide the pilot as he or she steps down the jet route.

3. The speed of Flight NAL63 is 1/3 foot/second. Stand at the beginning of the jet route for Flight NAL63. Place a mark (or a piece of masking tape) every 1/3 foot (every 4 inches) along the jet route. This will guide the pilot as he or she steps down the jet route.

4. On each jet route, place and label a longer chalk mark (or a longer piece of masking tape) at the following positions:
   5 feet from the start, **10 feet** from the start, **15 feet** from the start, the **finish point**
   Note: The finish point is where the jet routes meet.



Name

Conduct the Experiment

1. **Review your prediction.**

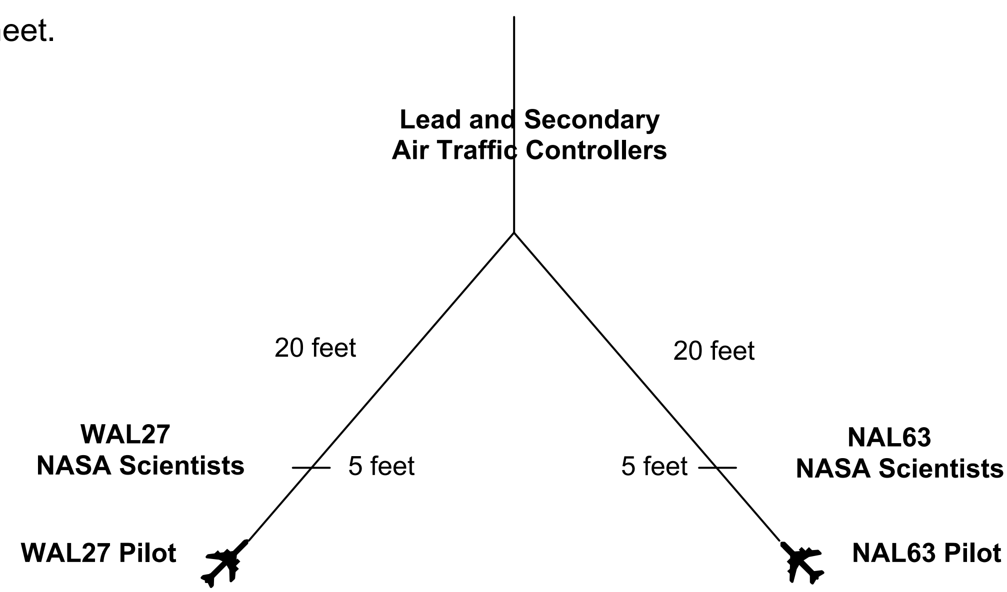
   Do you think the airplanes will meet at the point where the two routes meet? Why or why not? If not, how many feet apart do you think the planes will be when the first plane reaches the point where the routes come together?

2. **Take your position.** Circle your role in the diagram and in the following list:

   Lead Air Traffic Controller: Give the command "Take your ready positions."
   Pilots: Position yourself at the start of your jet route.
   Secondary Controller: Take your data sheet, measuring tape, and pencil and go to your controller location as shown below.
   NASA Scientists: Take your data sheet and pencil and go to your first observation position at the 5-foot line as shown below.

3. **Get ready to begin.** Circle your role in the following list:

   Lead Air Traffic Controller: Give the command "Set."
   Pilots: Prepare to step down your jet route. You may want to practice first. It helps to keep one foot on each side of the jet route.
   NASA Scientists: Get ready to measure and record the information on the data sheet.



____

Name

4. **Begin the experiment.** Circle your role in the following list:

   Lead Air Traffic Controller: Give the command “Ready.” Start your stopwatch and count the seconds aloud, “One, two, three…” and so on.

   Pilots: Take your first step on count “One.” Each second, take one step to the next timing mark.

   NASA Scientists: Record the time your aircraft arrives at the 5-foot line. Stay ahead of the pilot and record the time your aircraft arrives at the 10-foot line, the 15-foot line, and the point where the Controller says, “Halt.”

5. **End the experiment.** Circle your role in the following list:

   Secondary Controller: When the first Pilot reaches the point where the two routes meet, give the command “Halt.” Measure and record the separation distance between the planes. To do this, measure the distance of the second Pilot from the point where the two routes meet.

   Lead Air Traffic Controller: Stop counting the seconds when you hear “Halt.”

   Pilots: Stop and remain where you are on the jet route when you hear “Halt.”

   NASA Scientists: Record the “Halt” time.

Name ____________________

Data Sheet

a. Fill in this table:

Flight Number	Speed	Distance from the Point Where the Two Routes Meet
WAL27		
NAL63		

b. On the picture below, circle your job title. Notice the data you need to record.

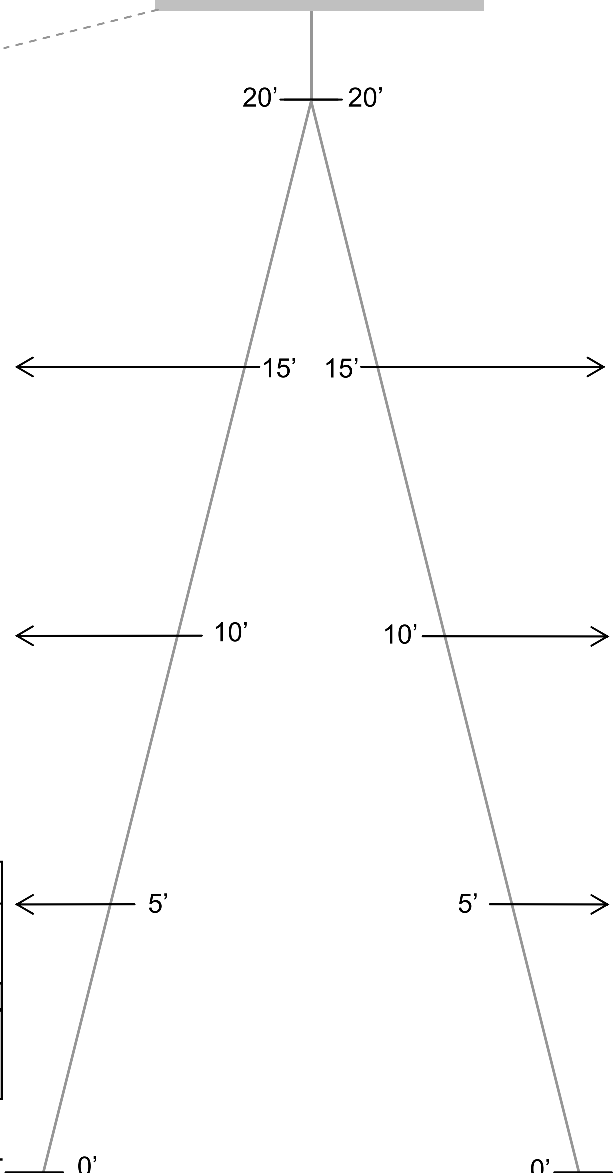
c. During Experiments 1, 2, and 3, record your data.

1	**2**	**3**

Separation Distance (feet) at Intersection

1	2	3

Air Traffic Controllers



"HALT" Time (WAL27 side)

1	2	3

Time (seconds)

1	2	3

Time (seconds)

1	2	3

Time (seconds)

1	2	3

NASA Scientist: WAL27
Start recording here.

"HALT" Time (NAL63 side)

1	2	3

Time (seconds)

1	2	3

Time (seconds)

1	2	3

Time (seconds)

1	2	3

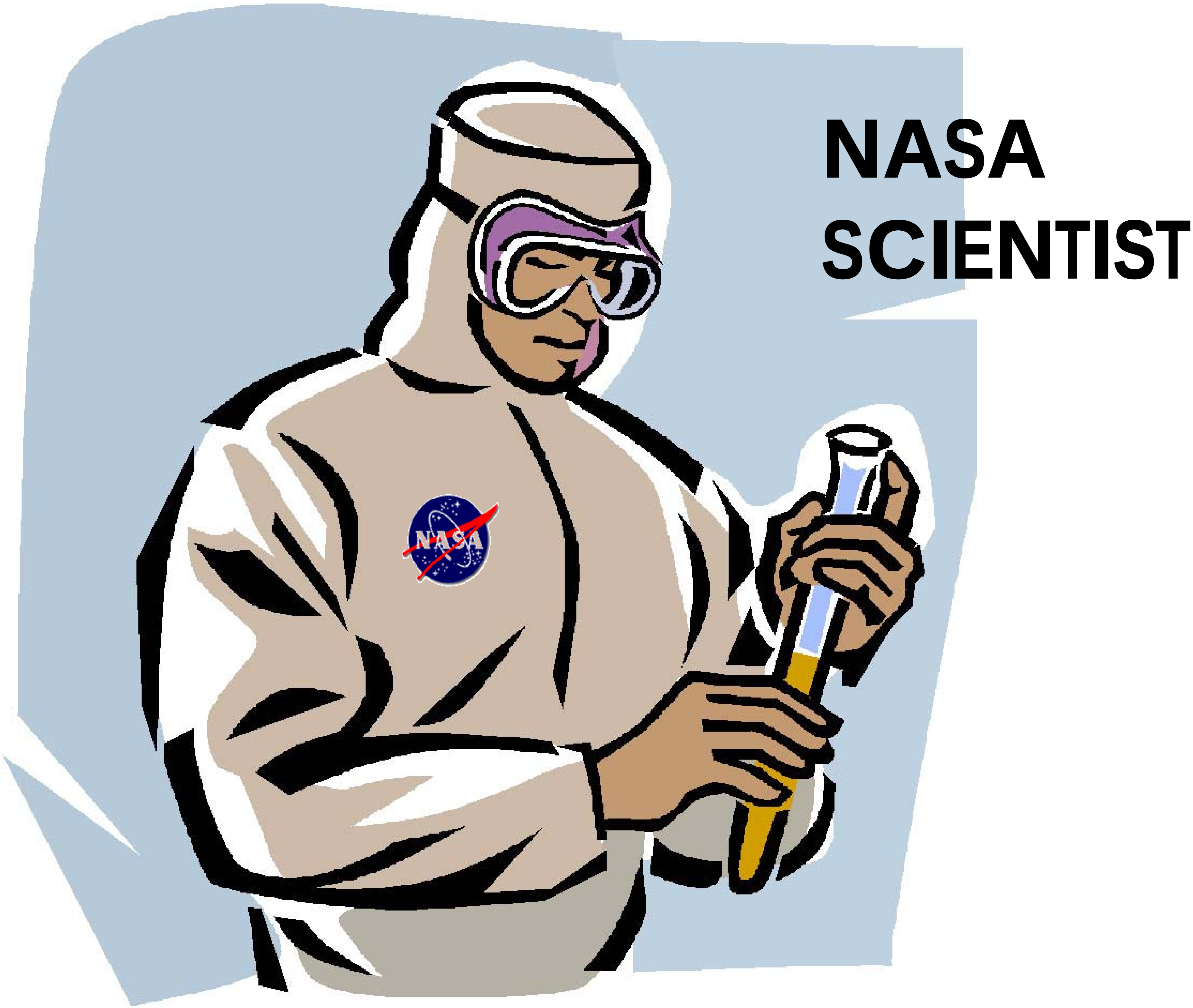
NASA Scientist: NAL63
Start recording here.

Pilot: WAL27

Pilot: NAL63

PILOT

AIR TRAFFIC CONTROLLER

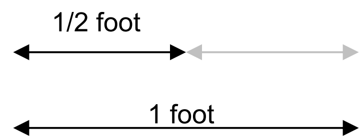

NASA
SCIENTIST

____
Name

How Much Time To Reach the Point Where the Two Routes Meet?
(Count Feet and Seconds to Find the Answer)

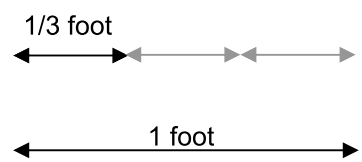
The speed of Flight WAL27 is 1/2 foot per second.
That means the plane travels 1/2 foot in 1 second.

So Flight WAL27 travels **1 foot in 2 seconds**.



The speed of Flight NAL63 is 1/3 foot per second.
That means the plane travels 1/3 foot in 1 second.

So Flight NAL63 travels **1 foot in 3 seconds**.



Flight WAL27 starts 20 feet from the point where the two routes meet.
Flight NAL63 starts 20 feet from the point where the two routes meet.

1. Fill in the given table to see how many seconds it will take each plane to travel to the point where the two routes meet.

After you fill in the table, answer the following questions:

2. How many seconds will it take each plane to arrive at the point where the two routes meet?

WAL27 ____________ seconds NAL63 ____________ seconds

____
Name

3. Will the two planes meet at the point where the two routes come together? _______________

4. Why or why not? ____
____

5. If you think the planes will not meet, how many feet apart will the planes be when the first plane reaches the point where the routes come together? _______________

6. Does your answer to Question 5 meet the 5-foot separation standard? That is, do you think the planes will be at least 5 feet apart when the first plane reaches the point where the routes come together? _______________

7. You filled in the table to find the answers. Can you think of a faster way to find the answer? If so, describe the faster way. _______________________
____

8. If you think the two planes will *not* meet the 5-foot separation standard, what could you tell the air traffic controllers to do to make sure that the separation standard will be met? ______________________________________
____

Name

Flight WAL27		Flight NAL63	
How many feet?	How many seconds?	How many feet?	How many seconds?
1	2	1	3
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			

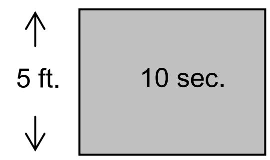
Name

How Much Time To Reach the Point Where the Two Routes Meet?
(Draw Blocks to Find the Answer)

Each plane starts 20 feet from the point where the two routes meet.

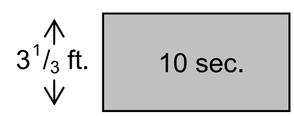
The speed of Flight WAL27 is 1/2 foot per second.
In 1 second, the plane travels 1/2 foot.
In 10 seconds, the plane travels 10 x 1/2 foot.
That is, Flight WAL27 travels 5 feet in 10 seconds.

The height of this block represents 5 feet, the distance Flight WAL27 travels in 10 seconds.



The speed of Flight NAL63 is 1/3 foot per second.
In 1 second, the plane travels 1/3 foot.
In 10 seconds, the plane travels 10 x 1/3 foot.
That is, Flight NAL63 travels $3^{1}/_{3}$ feet in 10 seconds.

The height of this block represents $3^{1}/_{3}$ feet, the distance Flight NAL63 travels in 10 seconds.



Now you will use blocks and dots to plot the position of each plane as it travels to where the two routes meet.

Look at the top picture on page 4. It shows the two jet routes.

Flight NAL63 started 20 feet away from the point where the two routes meet.
After 10 seconds, Flight NAL63 has moved $3^{1}/_{3}$ feet closer to that point.
So the plane is $16^{2}/_{3}$ feet from the point where the two routes meet.
A dot shows the position of Flight NAL63 after 10 seconds.

Name

Flight WAL27 started 20 feet away from the point where the two routes meet.

After 10 seconds, Flight WAL27 has moved 5 feet closer to that point.

So the plane is 15 feet from the point where the two routes meet.

A dot shows the position of Flight WAL27 after 10 seconds.

The dots are connected with a line marked “10 seconds.”

Next look at the bottom picture on page 4. The picture shows a bar graph made of blocks.

A block shows the position of Flight NAL63 after 10 seconds.

A block shows the position of Flight WAL27 after 10 seconds.

The blocks are connected with a line marked “10 seconds.”

Now it’s your turn to draw and connect.

- □ On the top picture on page 4, draw a dot to show the position of Flight NAL63 after 20 seconds.
- □ On the bottom picture on page 4, trace the block that shows the position of Flight NAL63 after 20 seconds.

- □ On the top picture, draw a dot to show the position of Flight WAL27 after 20 seconds.
- □ On the bottom picture, trace the block that shows the position of Flight WAL27 after 20 seconds.

- □ Connect your dots with a line marked “20 seconds.”
- □ Connect your blocks with a line marked “20 seconds.”

- □ Draw and connect dots at 30 seconds.
- □ Draw and connect blocks at 30 seconds.

- □ Keep going until the first plane reaches the place where the two routes meet.

____
Name

When you are done, answer these questions.

1. Circle the flight number of the first plane to arrive at the point where the two routes meet. How many seconds will it take that plane to arrive at the point where the two routes meet?

   WAL27 ____________ seconds NAL63 ____________ seconds

2. Will the two planes meet at the point where the two routes come together? ________________

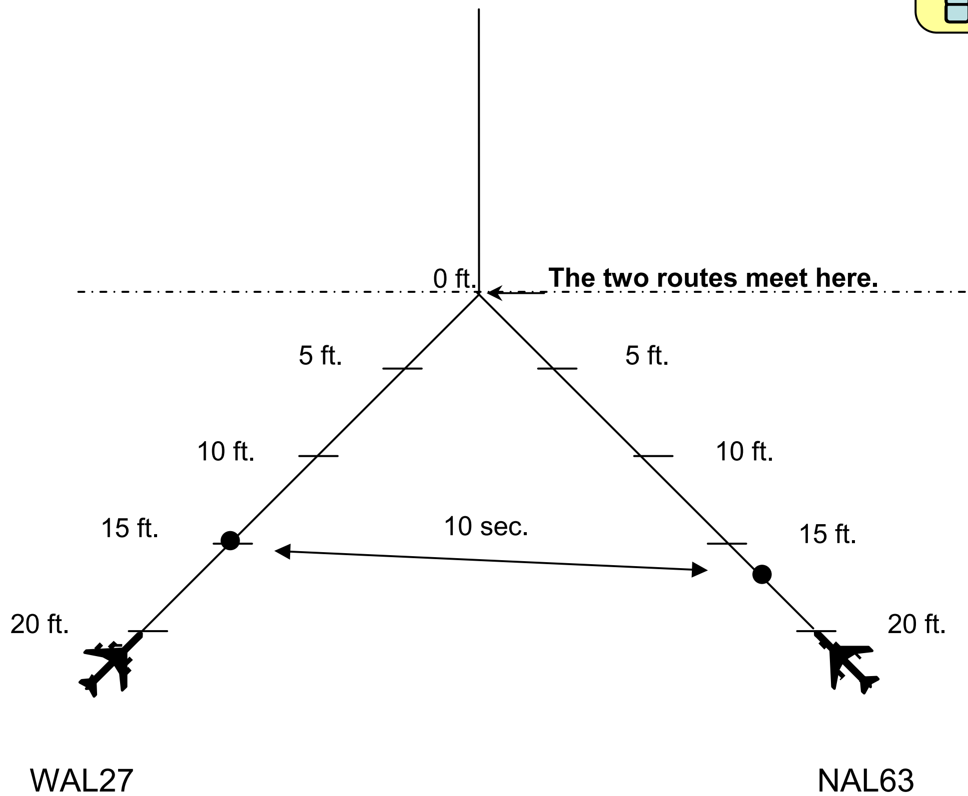
3. Why or why not? ____

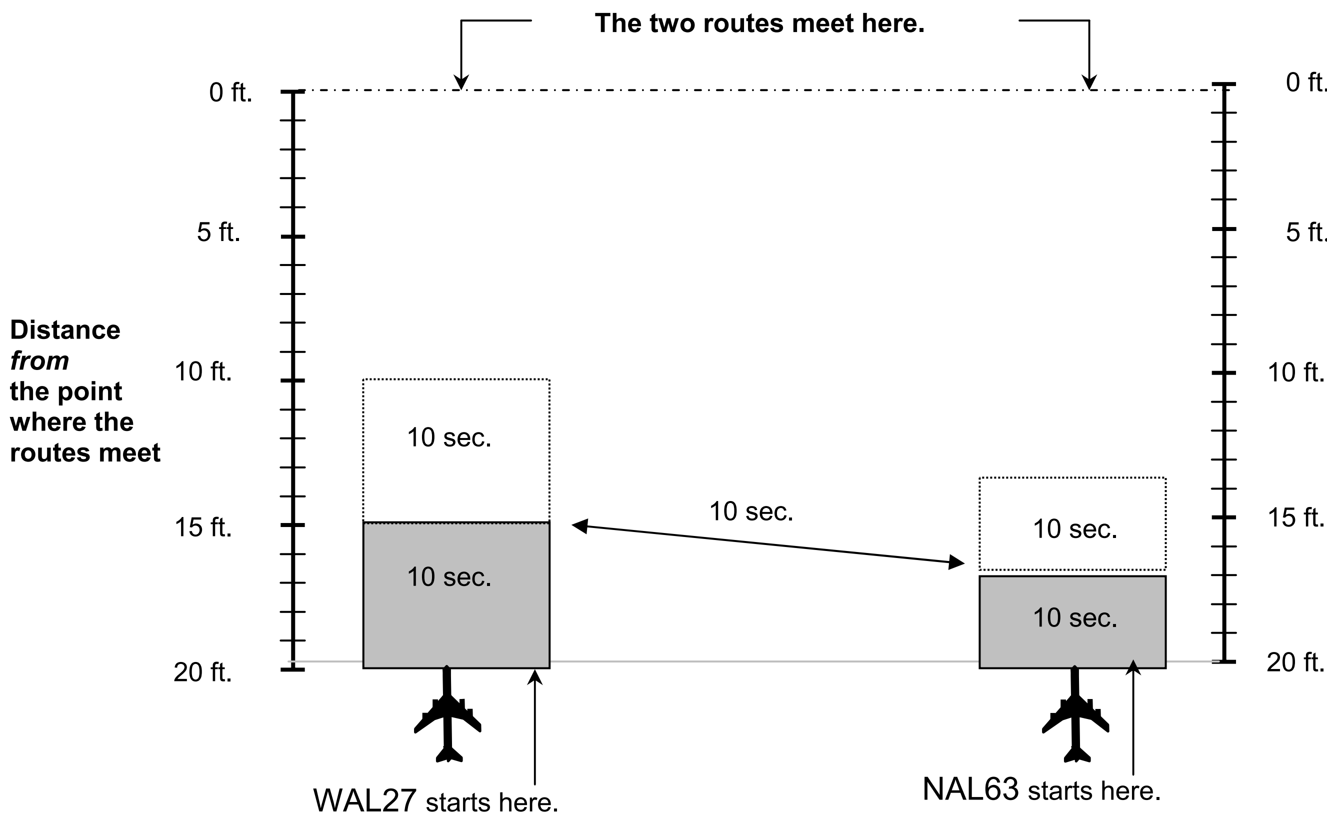
4. If you think the planes will not meet, how many feet apart will the planes be when the first plane reaches the point where the routes come together? _________________

5. Does your answer to Question 4 meet the 5-foot separation standard? That is, do you think the planes will be at least 5 feet apart when the first plane reaches the point where the routes come together? ________________

6. If you think the two planes will *not* meet the 5-foot separation standard, what could you tell the air traffic controllers to do to make sure that the separation standard will be met? ____
____

Name


0 ft.
The two routes meet here.
5 ft.
5 ft.
10 ft.
10 ft.
15 ft.
10 sec.
15 ft.
20 ft.
20 ft.
WAL27
NAL63


The two routes meet here.
0 ft.
0 ft.
5 ft.
5 ft.
Distance *from* the point where the routes meet
10 ft.
10 ft.
10 sec.
10 sec.
15 ft.
15 ft.
10 sec.
10 sec.
10 sec.
20 ft.
20 ft.
WAL27 starts here.
NAL63 starts here.

____
Name

How Much Time To Reach the Point Where the Two Routes Meet?
(Plot Points on Lines to Find the Answer)

Each flight starts 20 feet from the point where the two routes meet.

The speed of Flight WAL27 is 1/2 foot per second.
In 1 second, the plane travels 1/2 foot.
In 10 seconds, the plane travels 10 x 1/2 foot.
That is, Flight WAL27 travels 5 feet in 10 seconds.

The speed of Flight NAL63 is 1/3 foot per second.
In 1 second, the plane travels 1/3 foot.
In 10 seconds, the plane travels 10 x 1/3 foot.
That is, Flight WAL27 travels $3\,^{1}/_{3}$ feet in 10 seconds.

You will use an **O** to show the position of Flight NAL63.
You will use an **X** to show the position of Flight WAL27.

Look at the top picture on page 4. The picture shows the two jet routes.

Flight NAL63 started 20 feet away from the point where the two routes meet.
After 10 seconds, Flight NAL63 has moved $3\,^{1}/_{3}$ feet closer to that point.
So the plane is $16\,^{2}/_{3}$ feet from the point where the two routes meet.
An **O** shows the position of Flight NAL63 after 10 seconds.

Flight WAL27 started 20 feet away from the point where the two routes meet.
After 10 seconds, Flight WAL27 has moved 5 feet closer to that point.
So the plane is 15 feet from the point where the two routes meet.
An **X** shows the position of Flight WAL27 after 10 seconds.

The **X** and **O** are connected with a line marked “10 seconds.”

Name

Next look at the bottom picture on page 4. The picture shows two vertical line graphs.

An **O** shows the position of Flight NAL63 after 10 seconds.
An **X** shows the position of Flight WAL27 after 10 seconds.
The **X** and **O** are connected with a line marked “10 seconds.”

Now it’s your turn to draw and connect.

- ◻ On the top picture on page 4, draw an **O** to show the position of Flight NAL63 after 20 seconds.
- ◻ On the bottom picture on page 4, draw an **O** to show the position of Flight NAL63 after 20 seconds.

- ◻ On the top picture, draw an **X** to show the position of Flight WAL27 after 20 seconds.
- ◻ On the bottom picture, draw an **X** to show the position of Flight WAL27 after 20 seconds.

- ◻ On the top picture, connect the **X** and **O** with a line marked “20 seconds.”
- ◻ On the bottom picture, do the same thing.

- ◻ On the top picture, draw and connect an **X** and an **O** at 30 seconds.
- ◻ On the bottom picture, do the same thing.

- ◻ Keep going until the first plane reaches the place where the two routes meet.

____
Name

When you are done, answer these questions.

1. Circle the flight number of the first plane to arrive at the point where the two routes meet. How many seconds will it take that plane to arrive at the point where the two routes meet?

   WAL27 ____________ seconds   NAL63 ____________ seconds

2. Will the two planes meet at the point where the two routes come together? ________________

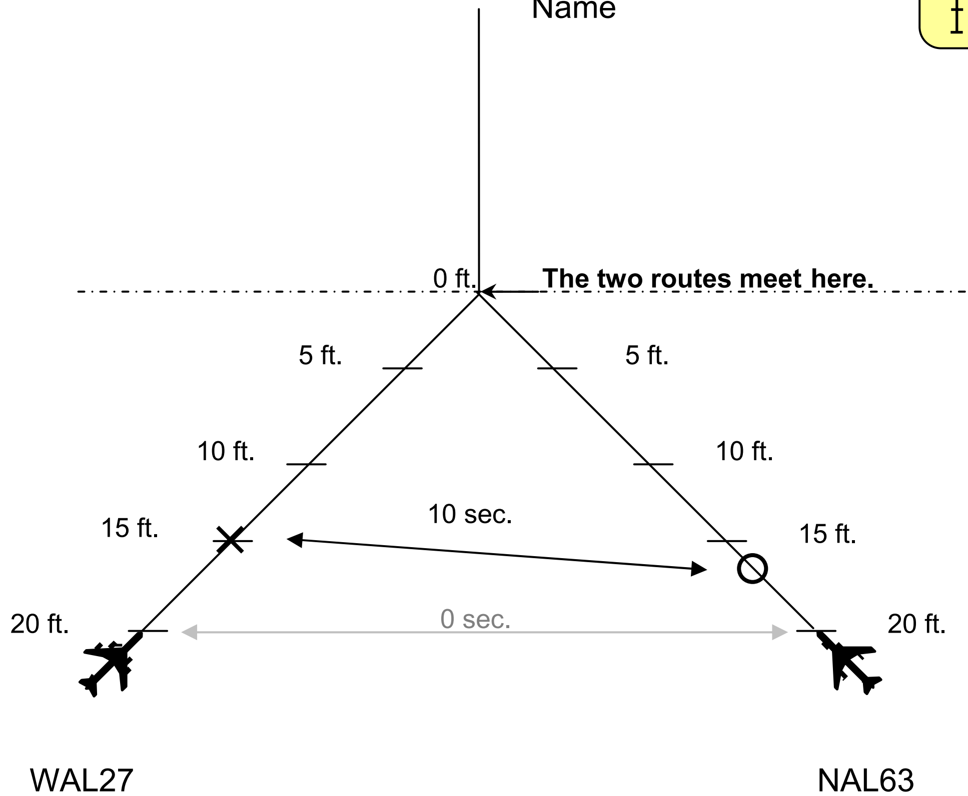
3. Why or why not? ____

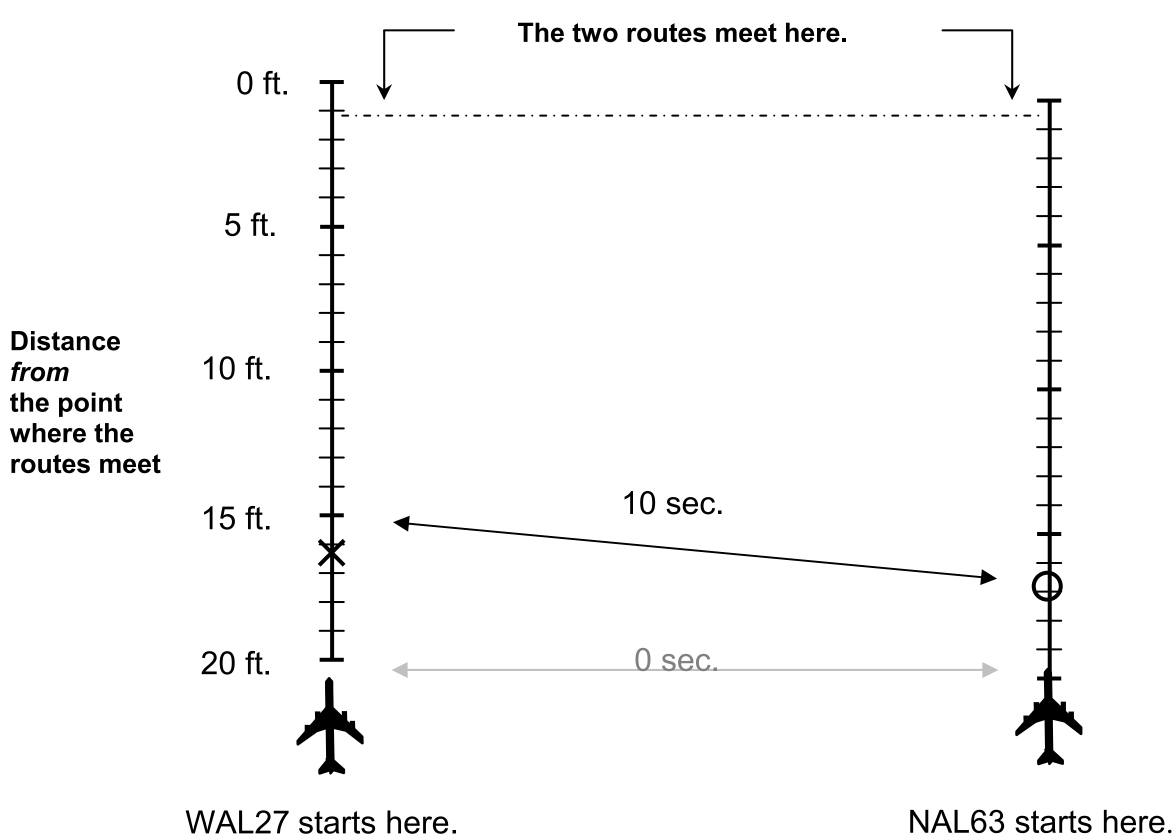
4. If you think the planes will not meet, how many feet apart will the planes be when the first plane reaches the point where the routes come together? _________________

5. Does your answer to Question 4 meet the 5-foot separation standard? That is, do you think the planes will be at least 5 feet apart when the first plane reaches the point where the routes come together? ________________

6. If you think the two planes will *not* meet the 5-foot separation standard, what could you tell the air traffic controllers to do to make sure that the separation standard will be met? ____
   ____

Name


0 ft.
The two routes meet here.
5 ft.
5 ft.
10 ft.
10 ft.
15 ft.
10 sec.
15 ft.
20 ft.
0 sec.
20 ft.
WAL27
NAL63


The two routes meet here.
0 ft.
5 ft.
Distance *from* the point where the routes meet
10 ft.
15 ft.
10 sec.
20 ft.
0 sec.
WAL27 starts here.
NAL63 starts here.

Name

How Much Time To Reach the Point Where the Routes Meet?
(Plot Points on a Grid to Find the Answer)

The speed of Flight WAL27 is 1/2 foot per second.
In 1 second, the plane travels 1/2 foot.
In 10 seconds, Flight WAL27 travels 5 feet.
The speed of Flight NAL63 is 1/3 foot per second.
In 1 second, the plane travels 1/3 foot.
In 10 seconds, Flight WAL27 travels $3\,^{1}/_{3}$ feet.

Flight NAL63 starts 20 feet from the point where the two routes meet.
After 10 seconds, the plane has traveled $3\,^{1}/_{3}$ feet.
So after **10** seconds, Flight NAL63 is **$16\,^{2}/_{3}$** feet ***from*** the point where the two routes meet.

On the NAL63 jet route on page 4, we represent that information with an **O** at $16\,^{2}/_{3}$ feet and an arrow labeled “10 seconds.”

On the grid on page 4, we represent that information with the point (**10**, **$-16\,^{2}/_{3}$**).
We use **negative** $16\,^{2}/_{3}$ because the point lies **below** the horizontal line at 0 feet where the two routes meet.
The **O** at the point (10, $-16\,^{2}/_{3}$) shows the position of Flight NAL63 after 10 seconds.

Flight WAL27 starts 20 feet from the point where the two routes meet.
After 10 seconds, the plane has traveled 5 feet.
So after **10** seconds, Flight WAL27 is **15** feet ***from*** the point where the two routes meet.

On the WAL27 jet route, we represent that information with an **X** at 15 feet and an arrow labeled “10 seconds.”

Name

On the grid, we represent that information with the point (**10**, **-15**).

We use **negative** fifteen because the point lies **below** the horizontal line at 0 feet where the two routes meet.

The **X** at the point (10, -15) shows the position of Flight WAL27 after 10 seconds.

Now it's your turn to plot and connect points on the routes and to plot point on the grid on page 4.

Put an **O** at the point that shows the position of Flight NAL63 after 20 seconds, 30 seconds and so on.

Put an **X** at the point that shows the position of Flight WAL27 after 20 seconds, 30 seconds, and so on.

Keep going until one plane reaches the horizontal line at 0 ft. where the two routes meet.

When you have finished plotting points, answer these questions.

1. Circle the flight number of the first plane to arrive at the point where the two routes meet. How many seconds will it take that plane to arrive at the point where the two routes meet?

   WAL27 ___________ seconds NAL63 ___________ seconds

2. Will the two planes meet at the point where the two routes come together? ______________

____
Name

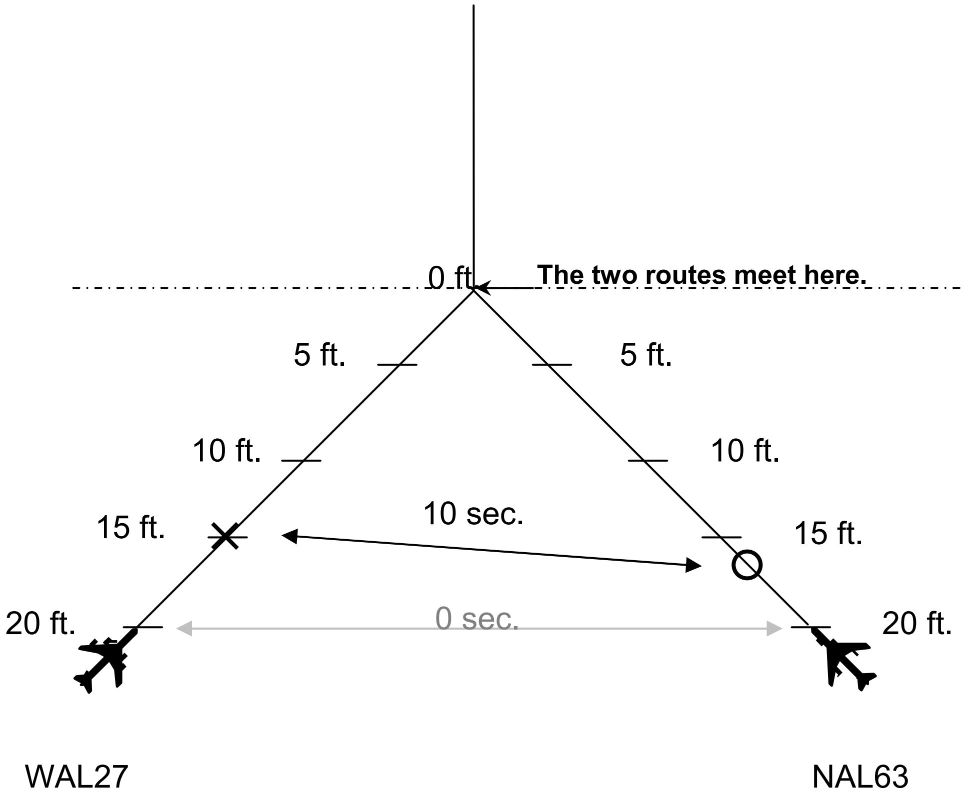
3. Why or why not? ____

4. If you think the planes will not meet, how many feet apart will the planes be when the first plane reaches the point where the routes come together? ________________

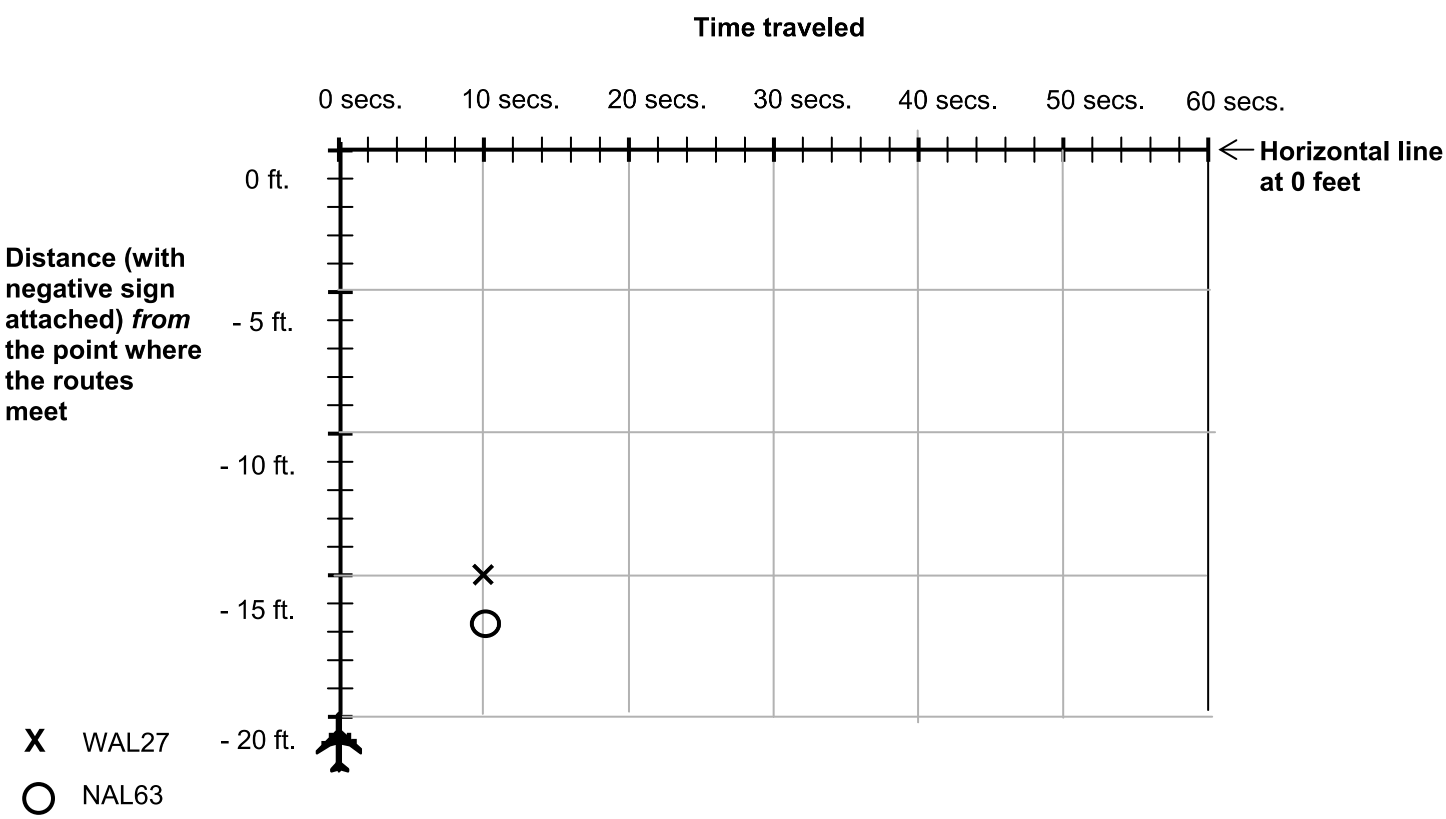
5. Does your answer to Question 4 meet the 5-foot separation standard? That is, do you think the planes will be at least 5 feet apart when the first plane reaches the point where the routes come together? ________________

6. If you think the two planes will *not* meet the 5-foot separation standard, what could you tell the air traffic controllers to do to make sure that the separation standard will be met? ____
____

Name



0 ft.
The two routes meet here.
5 ft.
5 ft.
10 ft.
10 ft.
10 sec.
15 ft.
15 ft.
20 ft.
0 sec.
20 ft.
WAL27
NAL63



Time traveled
0 secs.
10 secs.
20 secs.
30 secs.
40 secs.
50 secs.
60 secs.
Horizontal line at 0 feet
0 ft.
Distance (with negative sign attached) *from* the point where the routes meet
- 5 ft.
- 10 ft.
- 15 ft.
- 20 ft.
X WAL27
O NAL63

____
Name

d = r • t

Derive the Distance-Rate-Time Formula

The speed of Flight WAL27 is 0.5 feet per second.

In 1 second, the plane travels 0.5 feet.

In 2 seconds, the plane travels 0.5 feet/second $\times$ 2 seconds = 1.0 foot.

In 3 seconds, the plane travels 0.5 feet/second $\times$ 3 seconds = 1.5 feet.

1. In 4 seconds, the plane travels _____________ $\times$ _________ = ____ feet.
2. In 5 seconds, the plane travels _____________ $\times$ _________ = ____ feet.
3. In 6 seconds, the plane travels _____________ $\times$ _________ = ____ feet.

4. How could you use multiplication to find the distance Flight WAL27 travels in 14 seconds?

____

____

One way to find the distance is to multiply the plane's speed by 14 seconds, like this:

0.5 feet/second $\times$ 14 seconds = 7 feet

This suggests the following rule:

To find the distance traveled, multiply the speed by the time traveled.

In mathematics and science, we often say "rate" instead of "speed."

Then we can write the rule like this:

distance = **r**ate $\times$ **t**ime

This relationship is called the Distance-Rate-Time Formula. We often write it like this:

d = r • t

5. Use the formula to find the distance traveled by Flight WAL27 in 30 seconds.
   In 30 seconds, Flight WAL27 travels ___________ feet.

Name

d = r • t

6. The speed of Flight NAL63 is 1/3 foot per second.

   Use the formula **d = r • t** to find the distance traveled by Flight NAL63 in 30 seconds.

   In 30 seconds, Flight NAL63 travels __________ feet.

____
Name

t = d / r

Use the Distance-Rate-Time Formula

The speed of Flight WAL23 is 0.5 feet per second.

In 1 second, the plane travels 0.5 feet.

In 2 seconds, the plane travels 0.5 feet/second $\times$ 2 seconds = 1.0 foot.

In 3 seconds, the plane travels 0.5 feet/second $\times$ 3 seconds = 1.5 feet.

To find the distance traveled after t seconds, we multiply the rate by the time:

distance traveled = **r**ate of travel $\times$ **t**ime traveled

This relationship is called the Distance-Rate-Time Formula. We often write it like this:

$$\mathbf{d = r \bullet t}$$

If we divide both sides of this equation by r, we get a formula for time traveled:

$$\mathbf{t = \frac{d}{r}}$$

$$\frac{d}{r} = \frac{r \bullet t}{r}$$

You can use this formula to find the number of seconds for Flight WAL27 to travel 20 feet to the point where the two routes meet.

Flight WAL27 $\quad t = \dfrac{\text{20 feet}}{\text{0.5 feet per second}} =$ ______ seconds

(Hint: Divide 20 by 0.5 .)

Use the same formula to find the number of seconds for Flight NAL63 to travel 20 feet to the point where the two routes meet.

(Hint: The speed of Flight NAL63 is $\frac{1}{3}$ foot per second.)

Flight NAL63 ______ seconds

Name ______________________

t = d / r

Now answer these questions.

1. How many seconds will it take each plane to arrive at the point where the two routes meet?

   WAL27 ___________ seconds   NAL63 ___________ seconds

2. Will the two planes meet at the point where the two routes come together? _______________

3. Why or why not? ____

4. If you think the planes will not meet, how many feet apart will the planes be when the first plane reaches the point where the routes come together? ________________

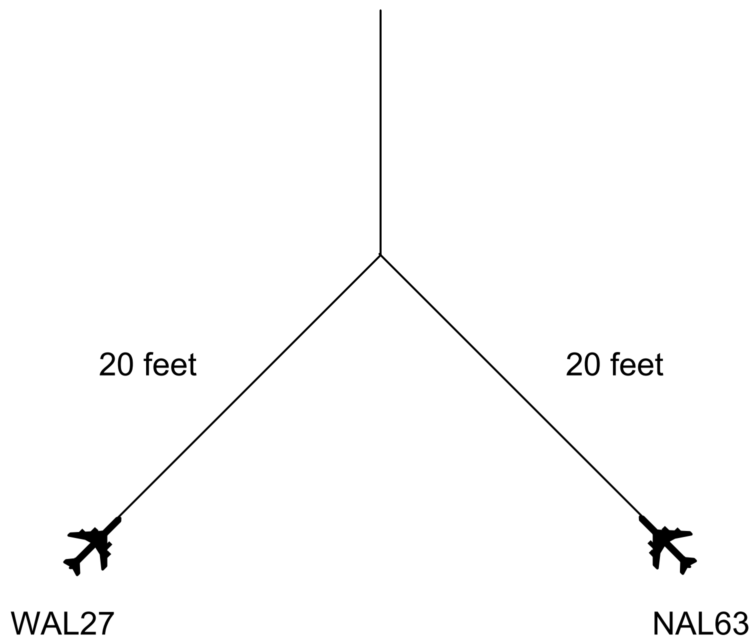
5. Does your answer to Question 4 meet the 5-foot separation standard? That is, do you think the planes will be at least 5 feet apart when the first plane reaches the point where the routes come together? _______________

6. If you think the two planes will *not* meet the 5-foot separation standard, what could you tell the air traffic controllers to do to make sure that the separation standard will be met? ____

   ____

Name

t = d / r



Name

How Much Time To Reach the Point Where the Two Routes Meet?
(Graph Two Linear Equations to Find the Answer)

We can use a linear equation to describe the position of an airplane that travels at a constant speed.

We begin with Flight WAL27.

The speed of Flight WAL27 is 1/2 foot per second.

When the clock starts at 0 seconds, the plane is 20 feet from the point where the two routes meet.

The position of Flight WAL27 is given by this equation:

$$y = (1/2)x - 20 \qquad \textbf{WAL27}$$

Here:

x = the time traveled (in seconds) and

y = the number of feet (with a negative sign attached) from the intersection of the two routes.

Notice that when $x = 0$, $y = -20$.

This means that when the clock starts at 0 seconds, the plane is 20 feet from the point where the two routes meet.

Now think about Flight NAL63.

The speed of Flight NAL63 is 1/3 foot per second. Its distance from the intersection is 20 feet. So the position of Flight NAL63 is given by this equation:

$$y = (1/3)x - 20 \qquad \textbf{NAL63}$$

Complete each table and use the ordered pairs to graph each line.

Name

WAL27 $y = (1/2)x - 20$

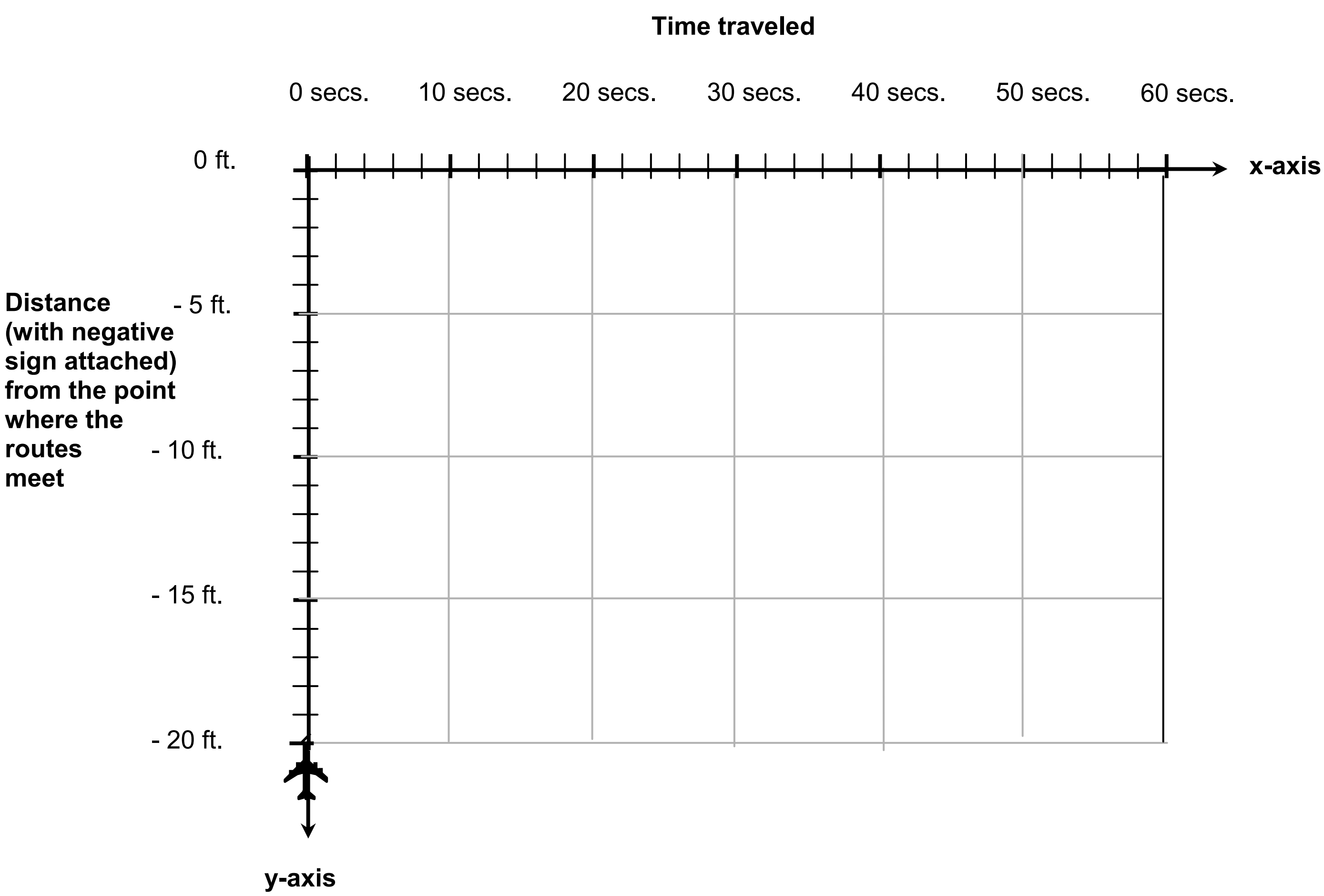
x	y
0	
10	
20	
30	
40	

NAL63 $y = (1/3)x - 20$

x	y
0	
10	
20	
30	
40	

Use a solid line for the graph of **Flight WAL27**.

Use a dotted line for the graph of **Flight NAL63**.



____
Name

Use your graphs to answer the following questions.

1. How many seconds will it take each plane to arrive at the point where the two routes meet?

   WAL27 ____________ seconds NAL63 ____________ seconds

2. Will the two planes meet at the point where the two routes come together? ________________

3. Why or why not? ____

4. If you think the planes will not meet, how many feet apart will the planes be when the first plane reaches the point where the routes come together? ________________

5. Does your answer to Question 4 meet the 5-foot separation standard? That is, do you think the planes will be at least 5 feet apart when the first plane reaches the point where the routes come together? ________________

6. If you think the two planes will *not* meet the 5-foot separation standard, what could you tell the air traffic controllers to do to make sure that the separation standard will be met? ____

   ____

7. Write the number that is the slope of the solid line representing Flight WAL27. ___________

8. Write the number that is the slope of the dotted line representing Flight NAL63. ___________

9. What information does the slope of each line tell you about each plane?

   ____

   ____

____
Name

After the Experiment

Now you will compare your prediction with the results of the experiment.

First, circle your role in the experiment:

Pilot of WAL27	NASA Scientist for WAL27	Lead Air Traffic Controller
Pilot of NAL63	NASA Scientist for NAL63	Other Air Traffic Controller

----

Take a look at your prediction.

Did you predict the planes would meet at the point where the two routes come together? Yes No

Next look at the results of the experiment.

Did the planes meet at the point where the two routes come together? Yes No

Does your prediction match the experiment? Yes No

If your answer to the last question is No, why do you think your prediction and the experiment do not match? ____

____

Take another look at your prediction.

When the first plane gets to the point where the routes meet, how many feet away did you think the second plane would be?

Does your prediction match the experiment?

If your answer is No, why do you think your prediction and the experiment do not match? ____

____

____
Name

Did the planes meet the 5-foot separation standard?
That is, were the planes at least 5 feet apart when the first plane reached the point where the routes come together? ________________

If planes did *not* meet the 5-foot separation standard, what could you tell the air traffic controllers to do to meet the separation standard next time?

____

____

Take another look at the problem.

The speed of Flight WAL27 was 1/2 foot per second.
The speed of Flight NAL63 was 1/3 foot per second.
Were the speeds the same or different?

Same speed Different speeds

Flight WAL27 started 20 feet from the point where the routes come together.
Flight NAL63 started 20 feet from the point where the routes come together.

What was the separation distance between the two planes when Flight NAL63 reached the point where the routes come together? ________________

How does the actual separation distance compare with the 5-foot separation standard?

____

____
Name

Now think about this general problem.

Two planes are traveling at different speeds on two different routes.
The planes are the same distance from the point where the two routes come together.

Will the planes meet at the point where the routes come together? ________________

Why or why not?____

____

Now suppose the difference in speeds is twice as great. What would you expect to happen to the separation distance at the point where the routes come together?
Why? ____

____
Name

Posttest

In the picture below, two airplanes are flying on different routes.

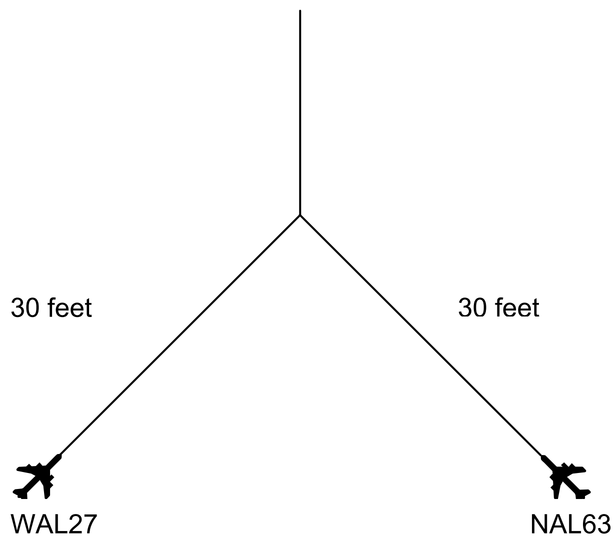
The speed of Flight WAL27 is 1 foot/second.

Flight WAL27 is 30 feet from the point where the two routes come together.

The speed of Flight NAL63 is 2/3 foot/second.

Flight NAL63 is 30 feet from the point where the two routes come together.

The separation standard is 5 feet.



1. Do you think that the two planes will meet at the point where the two routes come together? ________________

   Why or why not? ____

   ____

2. If not, how far apart do you think the planes will be when the first plane reaches the point where the routes come together? ________________

3. Does your answer to Question 2 meet the 5-foot separation standard? ________________

____
Name

4. If you think the two planes will *not* meet the 5-foot separation standard, what could you tell the air traffic controllers to do to make sure that the separation standard will be met? ____
____

Now consider this general problem.

Two planes are traveling at different speeds on two different routes.
The planes are the same distance from the point where the two routes come together.

5. Will the planes meet at the point where the routes come together?_____________

   Why or why not?

   ____
   ____

6. Now suppose the difference in speeds is twice as great. What would you expect to happen to the separation distance at the point where the routes come together? Why? ____

Airspace Systems – Predicting Air Traffic Conflicts

Curriculum Supplement 5

ANSWERS & EXPLANATIONS

A note on the organization of the Answers and Explanations:
In this Curriculum Supplement, most of the activities pose the same set of questions. The answers to those questions are introduced in the first part of this answer document. The remainder of the document is organized by activity and includes answers to individual activity questions, graphs, applications of the distance-rate-time formula, and discussions of the general problems posed in the analysis activity and the posttest.

The speed of Flight WAL27 is 1/2 foot/second, so the plane travels 1/2 foot in 1 second.
The speed of Flight NAL63 is 1/3 foot/second, so the plane travels 1/3 foot in 1 second.
Flight WAL27 is 20 feet from the point of intersection.
Flight NAL63 is 20 feet from the point of intersection.
A 5-foot separation distance is required. If the separation becomes less than 5 feet, then a separation violation will occur.

Since the planes are traveling at different constant (fixed) speeds and each must travel the same distance to the point of intersection, the planes will arrive at the intersection at different times. To determine whether or not a separation violation will occur at the intersection, students must calculate or graph to determine the number of seconds for the faster plane to reach the intersection and the number of feet between the two planes at that moment.

In particular:

- It will take 40 seconds for Flight WAL27 to travel 20 feet to the point where the routes come together.

- It will take 60 seconds for Flight NAL63 to travel 20 feet to the point where the routes come together.

- At 40 seconds, when Flight WAL27 arrives at the intersection, Flight NAL63 will be approximately 6.7 feet from the intersection. So the planes will be approximately 6.7 feet apart when the first plane arrives at the intersection and a separation violation will *not* occur.

Activity 5.3A—Count Feet and Seconds:

Flight WAL27 travels 1 foot in 2 seconds. Count by 2s to complete the table for Flight WAL 27.

Flight NAL63 travels 1 foot in 3 seconds. Count by 3s to complete the table for Flight NAL63.

Flight WAL27 will travel 20 feet in 40 seconds. (Students can also multiply 2 seconds per foot by 20 feet to obtain 40 seconds.)
Flight NAL63 will travel 20 feet in 60 seconds. (Students can also multiply 3 seconds per foot by 20 feet to obtain 60 seconds.)

So Flight WAL27 arrives at the intersection 20 seconds ahead of Flight NAL63.

To determine the separation distance between the planes, one must determine how far back Flight NAL63 is at the time the first plane arrives at the intersection. To do this, it is **critical** to use the speed of NAL63, which has not yet reached the intersection.

For Flight NAL63, every 3 seconds corresponds to 1 foot traveled. So the 20-second gap in time corresponds to 20 seconds divided by 3 seconds/foot. This is approximately 6.7 feet. This 6.7-foot separation is greater than the 5-foot separation standard. Therefore a separation violation will *not* occur.

1. Fill in the given table to see how many seconds it will take each plane to travel to the point where the two routes meet.

Flight WAL27		Flight NAL63	
How many feet?	How many seconds?	How many feet?	How many seconds?
1	2	1	3
2	**4**	**2**	**6**
3	**6**	**3**	**9**
4	**8**	**4**	**12**
5	**10**	**5**	**15**
6	**12**	**6**	**18**
7	**14**	**7**	**21**
8	**16**	**8**	**24**
9	**18**	**9**	**27**
10	**20**	**10**	**30**
11	**22**	**11**	**33**
12	**24**	**12**	**36**
13	**26**	**13**	**39**
14	**28**	**14**	**42**
15	**30**	**15**	**45**
16	**32**	**16**	**48**
17	**34**	**17**	**51**
18	**36**	**18**	**54**
19	**38**	**19**	**57**
20	**40**	**20**	**60**

Activity 5.3A (cont.)

2. How many seconds will it take each plane to arrive at the point where the two routes meet?

   WAL27 **40** seconds NAL63 **60** seconds

3. Will the two planes meet at the point where the two routes come together? **No**

4. Why or why not? **At 40 seconds, when Flight WAL27 arrives at the intersection, Flight NAL63 will be 20 seconds from the intersection. So the planes will not meet.**

5. If you think the two planes will not meet, how many feet apart will the planes be when the first plane reaches the point where the routes come together? **Approximately 6.7 feet.**

6. Does your answer to Question 5 meet the 5-foot separation standard? That is, do you think the planes will be at least 5 feet apart when the first plane reaches the point where the routes come together? **Yes.**

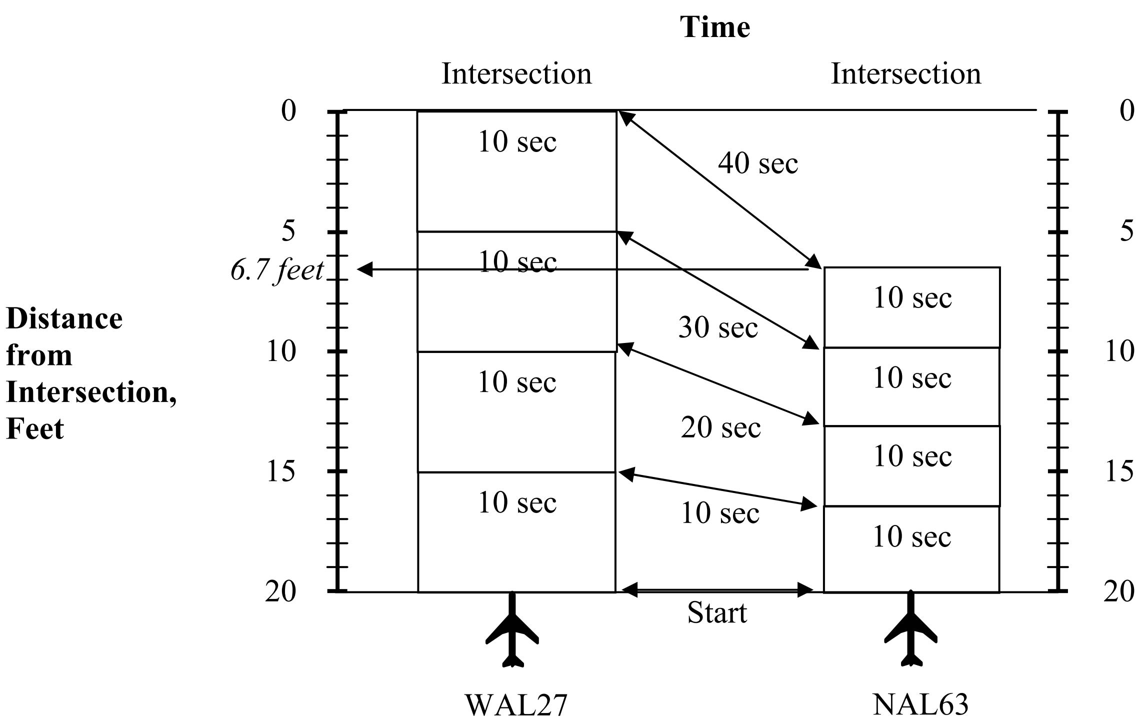
7. You filled in the table to find the answers. Can you think of a faster way to find the answer? If so, describe the faster way. **For Flight WAL27, multiply 2 seconds per foot by 20 feet to obtain 40 seconds. For Flight NAL63, multiply 3 seconds per foot by 20 feet to obtain 60 seconds.**

8. If you think the two planes will *not* meet the 5-foot separation standard, what could you tell the air traffic controllers to do to make sure that the separation standard will be met? **Change the speed or change the route of one of the planes.**

Activity 5.3B—Stacking Blocks:

Flight WAL27 travels 1 foot in 2 seconds. So in 10 seconds, Flight WAL27 will go 5 feet.
Flight NAL63 travels 1 foot in 3 seconds. So in 10 seconds, Flight NAL63 will go $3\,{}^{1}/_{3}$ feet (approximately 3.3 feet).
The following diagram shows a stack of 10-second blocks for each plane.
In the stack corresponding to Flight WAL27, each block represents 5 feet.
In the stack corresponding to Flight NAL63, each block represents approximately 3.3 feet.

As the blocks for Flight WAL27 are stacked, it will become clear that the flight arrives at the intersection in 40 seconds. At this time, Flight NAL63 is approximately 6.7 feet from the intersection. This distance is greater than the required 5-foot separation standard, so a separation violation will *not* occur.



1. Circle the flight number of the first plane to arrive at the point where the two routes meet. How many seconds will it take that plane to arrive at the point where the two routes meet?

   WAL27 __**40**__ seconds NAL63 __**60**__ seconds

2. Will the two planes meet at the point where the two routes come together? __**No**__

Activity 5.3B (cont.)

3. Why or why not? **At 40 seconds, when Flight WAL27 arrives at the intersection, Flight NAL63 will be 20 seconds from the intersection. So the planes will not meet.**

4. If you think the two planes will not meet, how many feet apart will the planes be when the first plane reaches the point where the routes come together? **Approximately 6.7 feet.**

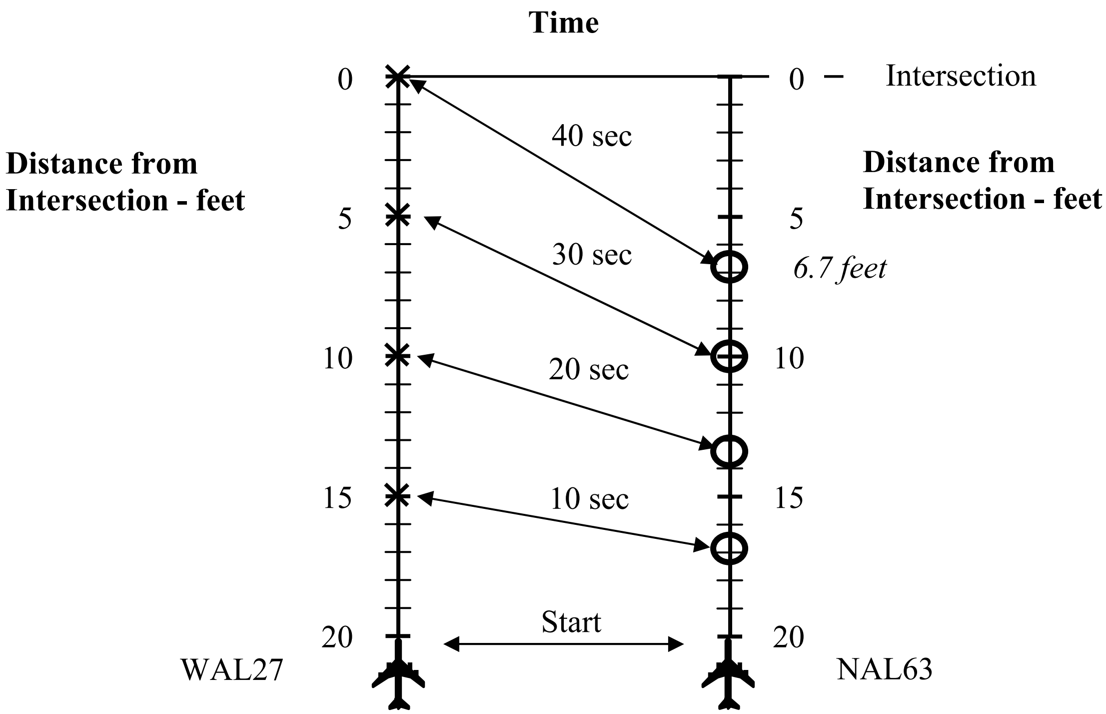
5. Does your answer to Question 4 meet the 5-foot separation standard? That is, do you think the planes will be at least 5 feet apart when the first plane reaches the point where the routes come together? **Yes.**

6. If you think the two planes will *not* meet the 5-foot separation standard, what could you tell the air traffic controllers to do to make sure that the separation standard will be met? **Change the speed or change the route of one of the planes.**

Activity 5.3C—Plot Points on Lines:

Flight WAL27 travels 1 foot in 2 seconds. So in 10 seconds, Flight WAL27 will go 5 feet. Flight NAL63 travels 1 foot in 3 seconds. So in 10 seconds, Flight NAL63 will go $3\,^1/_3$ feet (approximately 3.3 feet).
The following diagram shows the position of each plane at 10-second intervals.

As the points for Flight WAL27 are plotted, it will become clear that the flight arrives at the intersection in 40 seconds. At this time, Flight NAL63 is approximately 13.3 feet from the start and is therefore 6.7 feet from the intersection. This distance is greater than the required 5-foot separation standard, so a separation violation will *not* occur.



1. Circle the flight number of the first plane to arrive at the point where the two routes meet.
   How many seconds will it take that plane to arrive at the point where the two routes meet?

WAL27 ____**40**____ seconds NAL63 ____**60**____ seconds

2. Will the two planes meet at the point where the two routes come together? ____**No**____

Activity 5.3C (cont.)

3. Why or why not? **At 40 seconds, when Flight WAL27 arrives at the intersection, Flight NAL63 will be 20 seconds from the intersection. So the planes will not meet.**

4. If you think the two planes will not meet, how many feet apart will the planes be when the first plane reaches the point where the routes come together? **Approximately 6.7 feet.**

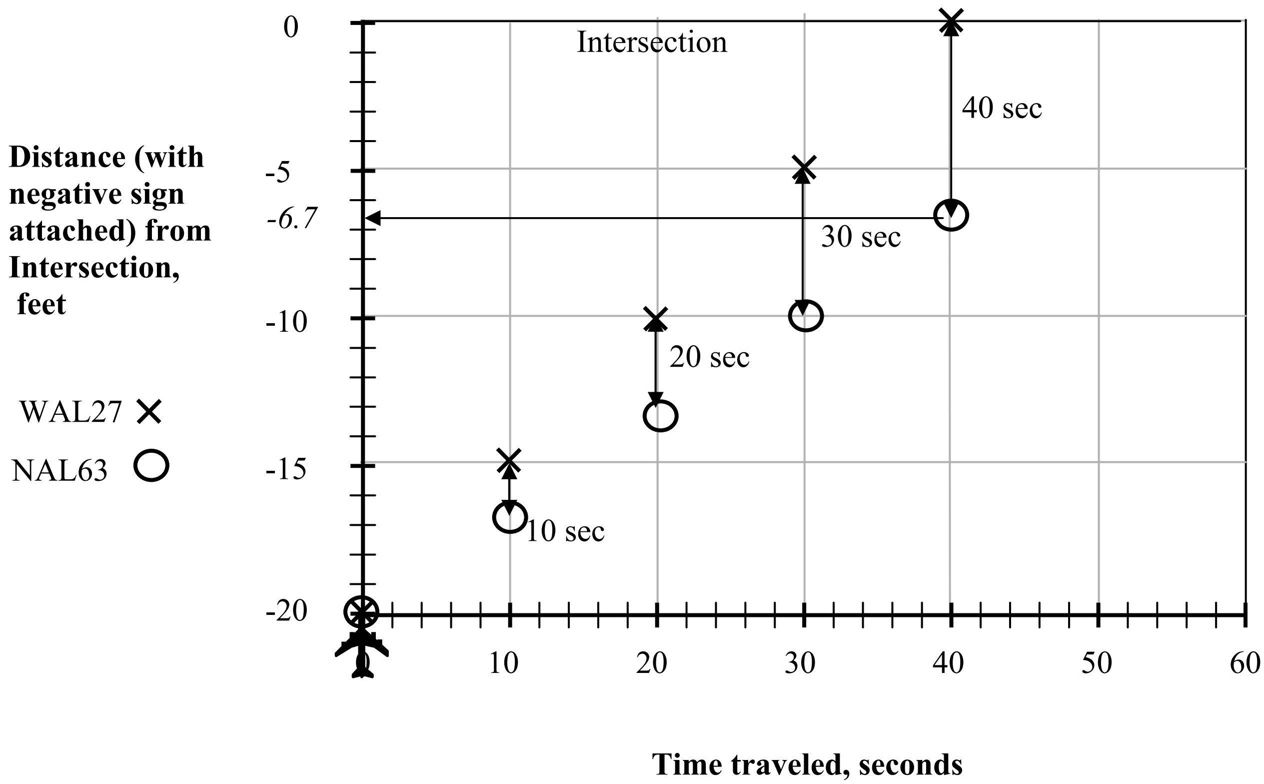
5. Does your answer to Question 4 meet the 5-foot separation standard? That is, do you think the planes will be at least 5 feet apart when the first plane reaches the point where the routes come together? **Yes.**

6. If you think the two planes will *not* meet the 5-foot separation standard, what could you tell the air traffic controllers to do to make sure that the separation standard will be met? **Change the speed or change the route of one of the planes.**

Activity 5.3D—Plot Points on a Grid:

Flight WAL27 travels 1 foot in 2 seconds. So in 10 seconds, Flight WAL27 will go 5 feet. Flight NAL63 travels 1 foot in 3 seconds. So in 10 seconds, Flight NAL63 will go $3\,^1/_3$ feet (approximately 3.3 feet).
The following graph shows the position of each plane at 10-second intervals.

As the points for Flight WAL27 are plotted, it will become clear that the flight arrives at the intersection in 40 seconds. At this time, Flight NAL63 is approximately 13.3 feet from the start and is therefore 6.7 feet from the intersection. This distance is greater than the required 5-foot separation standard, so a separation violation will *not* occur.



1. Circle the flight number of the first plane to arrive at the point where the two routes meet.
   How many seconds will it take that plane to arrive at the point where the two routes meet?

   WAL27 ___**40**___ seconds NAL63 ___**60**___ seconds

2. Will the two planes meet at the point where the two routes come together? __**No**__

Activity 5.3D (cont.)

3. Why or why not? **At 40 seconds, when Flight WAL27 arrives at the intersection, Flight NAL63 will be 20 seconds from the intersection. So the planes will not meet.**

4. If you think the two planes will not meet, how many feet apart will the planes be when the first plane reaches the point where the routes come together? **Approximately 6.7 feet.**

5. Does your answer to Question 4 meet the 5-foot separation standard? That is, do you think the planes will be at least 5 feet apart when the first plane reaches the point where the routes come together? **Yes.**

6. If you think the two planes will *not* meet the 5-foot separation standard, what could you tell the air traffic controllers to do to make sure that the separation standard will be met? **Change the speed or change the route of one of the planes.**

Activity 5.3E—Derive the Distance-Rate-Time Formula (Grades 6-8):

In 4 seconds, Flight WAL27 travels 0.5 feet/second x 4 seconds = 2.0 feet.

In 5 seconds, Flight WAL27 travels 0.5 feet/second x 5 seconds = 2.5 feet.

In 6 seconds, Flight WAL27 travels 0.5 feet/second x 5 seconds = 3.0 feet.

To find the distance Flight WAL27 travels in 14 seconds, multiply 0.5 feet/second by 14.

To find the distance Flight WAL27 travels in 30 seconds, multiply 0.5 feet/second by 30. The result is 15.0 feet.

To find the distance Flight NAL63 travels in 30 seconds, multiply 1/3 foot/second by 30. The result is 10 feet.

1. In 4 seconds, the plane travels **0.5 feet/second** × **4 seconds** = **2.0** feet.
2. In 5 seconds, the plane travels **0.5 feet/second** × **5 seconds** = **2.5** feet.
3. In 6 seconds, the plane travels **0.5 feet/second** × **6 seconds** = **3.0** feet.
4. How could you use multiplication to find the distance Flight WAL27 travels in 14 seconds? **Multiply 0.5 feet/second by 14 seconds.**
5. Use the formula to find the distance traveled by Flight WAL27 in 30 seconds.
   In 30 seconds, Flight WAL27 travels **15** feet.
6. The speed of Flight NAL63 is 1/3 foot per second.
   Use the formula **d = r • t** to find the distance traveled by Flight NAL63 in 30 seconds.
   In 30 seconds, Flight NAL63 travels **10** feet.

Activity 5.3F—Use the Distance-Rate-Time Formula:

Calculate the time it takes each aircraft to travel to the intersection.

The travel times are:

WAL27: t = 20 feet / 0.5 feet per second = 40 seconds

NAL63: t = 20 feet / (1/3) foot per second = 60 seconds

Flight NAL63 is 20 seconds behind Flight WAL27 when Flight WAL27 arrives at the intersection. To find the corresponding separation distance, one needs to know how far Flight NAL63 would travel in those 20 seconds to arrive at the intersection. To determine this, it is **critical** to use the speed of Flight NAL63, which has not yet reached the intersection.

Use the Distance-Rate-Time Formula in the form $d = r \cdot t$ with $r = 1/3$ foot per second and $t = 20$ seconds.

$$d = r \cdot t = (1/3) \text{ foot/second} \times 20 \text{ seconds} = 6\,{}^{2}/_{3} \text{ feet}$$

This distance, approximately 6.7 feet, is greater than the required 5-foot separation standard, so a separation violation will *not* occur.

Flight WAL27 $\quad t = \dfrac{20 \text{ feet}}{0.5 \text{ feet per second}} =$ __**40**__ seconds

(Hint: Divide 20 by 0.5 .)

Use the same formula to find the number of seconds for Flight NAL63 to travel 20 feet to the point where the two routes meet.

(Hint: The speed of Flight NAL63 is 1/3 foot per second.)

Flight NAL63 $\quad \mathbf{t} = \dfrac{\textbf{20 feet}}{\textbf{1/3 foot per second}} =$ __**60**__ seconds

1. How many seconds will it take each plane to arrive at the point where the two routes meet?

   WAL27 __**40**__ seconds NAL63 __**60**__ seconds

2. Will the two planes meet at the point where the two routes come together? __**No**__

Activity 5.3F (cont.)

3. Why or why not? **At 40 seconds, when Flight WAL27 arrives at the intersection, Flight NAL63 will be 20 seconds from the intersection. So the planes will not meet.**

4. If you think the two planes will not meet, how many feet apart will the planes be when the first plane reaches the point where the routes come together? **Approximately 6.7 feet.**

5. Does your answer to Question 4 meet the 5-foot separation standard? That is, do you think the planes will be at least 5 feet apart when the first plane reaches the point where the routes come together? **Yes.**

6. If you think the two planes will not meet the 5-foot separation standard, what could you tell the air traffic controllers to do to make sure that the separation standard will be met? **Change the speed or change the route of one of the planes.**

Activity 5.3G—Graph Two Linear Equations:

Use the time (40 seconds) when Flight WAL27 arrives at the intersection. On the line corresponding to Flight NAL63, locate the point where the time is 40 seconds. That point corresponds to a distance approximately 6.7 feet from the intersection. This distance is greater than the 5-foot separation standard, so a separation violation will *not* occur.

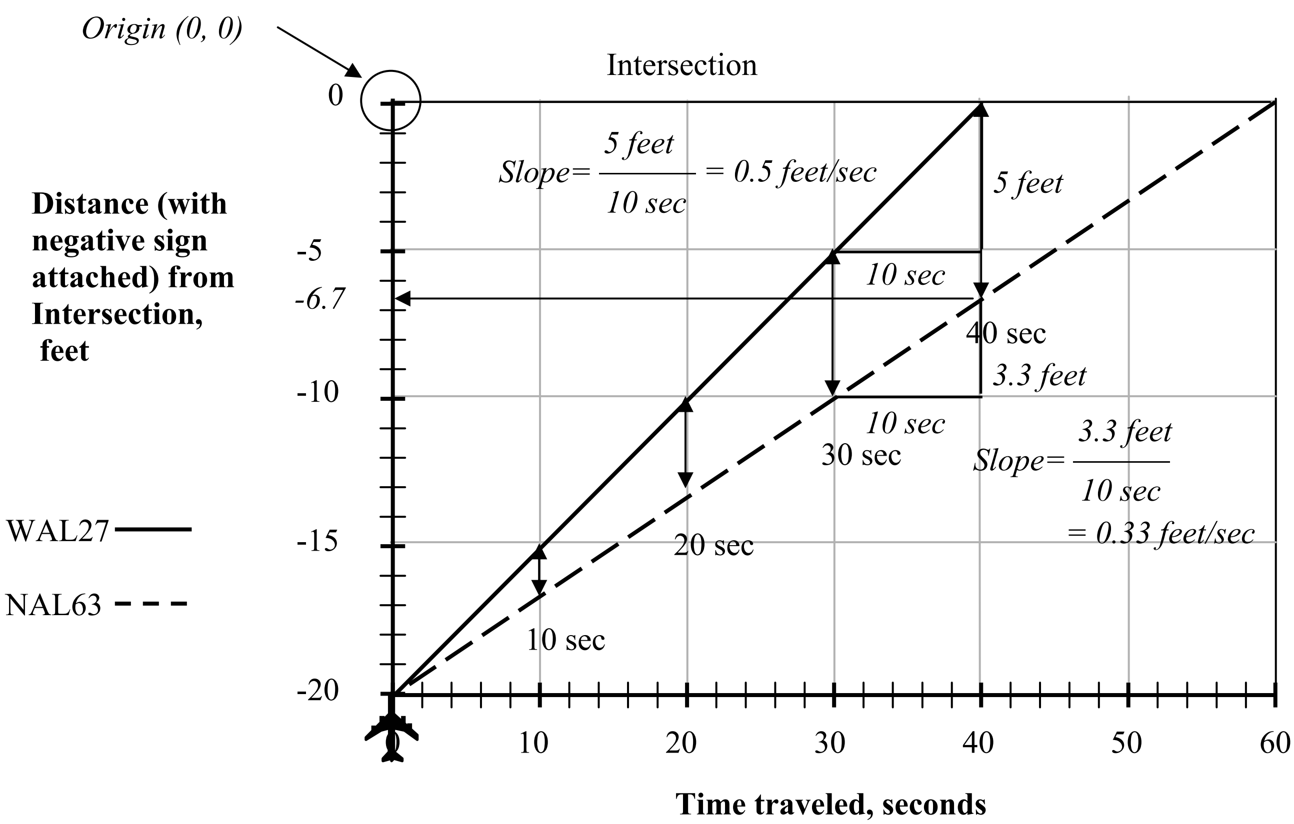
WAL27 $y = (1/2)x - 20$

x	y
0	**-20**
10	**-15**
20	**-10**
30	**-5**
40	**0**

NAL63 $y = (1/3)x - 20$

x	y
0	**-20**
10	**-16.7**
20	**-13.3**
30	**-10**
40	**-6.7**

For the line corresponding to Flight WAL27, the slope of the line is 0.5 feet/sec, the speed of Flight WAL27.
For the line corresponding to Flight NAL63, the slope of the line is approximately 0.33 feet/sec, the speed of Flight NAL63.
The lines have the same vertical intercept, -20. This corresponds to 20 feet, each plane's initial distance from the intersection.



Activity 5.3G (cont.)

1. How many seconds will it take each plane to arrive at the point where the two routes meet?

   WAL27 **40** seconds NAL63 **60** seconds

2. Will the two planes meet at the point where the two routes come together? **No**

3. Why or why not? **At 40 seconds, when Flight WAL27 arrives at the intersection, Flight NAL63 will be 20 seconds from the intersection. So the planes will not meet.**

4. If you think the two planes will not meet, how many feet apart will the planes be when the first plane reaches the point where the routes come together? **Approximately 6.7 feet.**

5. Does your answer to Question 4 meet the 5-foot separation standard? That is, do you think the planes will be at least 5 feet apart when the first plane reaches the point where the routes come together? **Yes.**

6. If you think the two planes will *not* meet the 5-foot separation standard, what could you tell the air traffic controllers to do to make sure that the separation standard will be met? **Change the speed or change the route of one of the planes.**

7. Write the number that is the slope of the solid line representing Flight WAL27. **1/2 foot/second**

8. Write the number that is the slope of the dotted line representing Flight NAL63. **1/3 foot/second**

9. What information does the slope of each line tell you about each plane?
   For Flight WAL27, the slope of the line is 1/2 foot/second, the speed of Flight WAL27. For Flight NAL63, the slope of the line is 1/3 foot/second, the speed of Flight NAL63.

Activity 5.4—After the Experiment and Activity 5.5—Posttest:

The plane speeds are different.
The planes are each the same distance from the finish point.

When Flight WAL27 reached the intersection, the separation distance was approximately 6.7 feet. This met the 5-foot separation standard.

General Problem:
Suppose two planes are traveling at different speeds on two different routes and the planes are each the same distance from the point where the two routes come together:

Since the planes are traveling at the different speeds and must each travel the same distance to the point of intersection, the planes will arrive at the intersection at different times.

If the difference in speeds were twice as great, then the separation distance at the intersection would also be twice as great. Students may understand this relationship based upon the activities they have completed.

To obtain a more formal explanation, one can use the Distance-Rate-Time formula as follows:

Let d_F, r_F, and t_F represent the distance, rate, and time of the Faster plane.
Let d_S, r_S, and t_S represent the distance, rate, and time of the Slower plane.

Examine the separation distance, $d_F - d_S$, at the time when the Faster plane reaches the intersection. That time is t_F.

At time t_F, the distance traveled by the Faster plane is: $d_F = r_F \cdot t_F$

At time t_F, the distance traveled by the Slower plane is: $d_S = r_S \cdot t_F$
(Notice that the time is the arrival time of the Faster plane.)

The separation distance is:

$$
\begin{aligned}
d_F - d_S &= r_F \cdot t_F - r_S \cdot t_F \\
&= t_F\,(r_F - r_S)
\end{aligned}
$$

So the separation distance is proportional to the difference in speeds.

If the difference in speeds is doubled, the new separation distance $= t_F \cdot 2(r_F - r_S)$

Therefore, the separation distance is doubled.